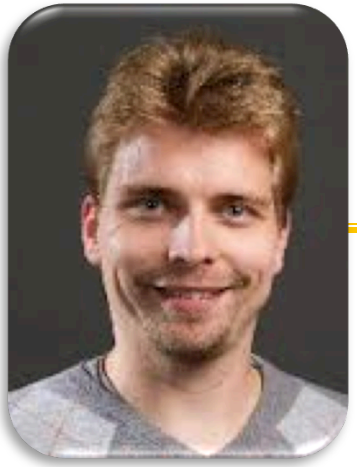

Computational Structures in Data Science

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Lecture #3: Recursion

Go watch Inception!

(Movie about recursion)

September 9, 2016

http://inst.eecs.berkeley.edu/~cs88

CS88 news

- Homework will have “Challenge problems”

- Project 1 coming soon!

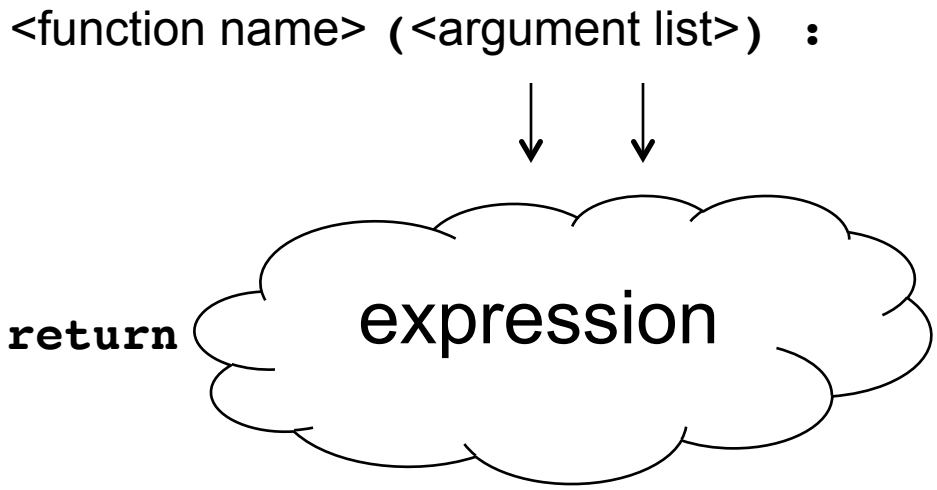
Site to know: www.stackoverflow.com

- Enrollment up to about 50. We might open a 3rd section.

Computational Concepts today

- Variable Scope (also: see reading)
- Recursion

Remember: Functions




```
def concat(str1, str2):
    return str1+str2;

concat("Hello","World")
```

- **Generalizes an expression or set of statements to apply to lots of instances of the problem**
- **A function should *do one thing well***

Variable Scope

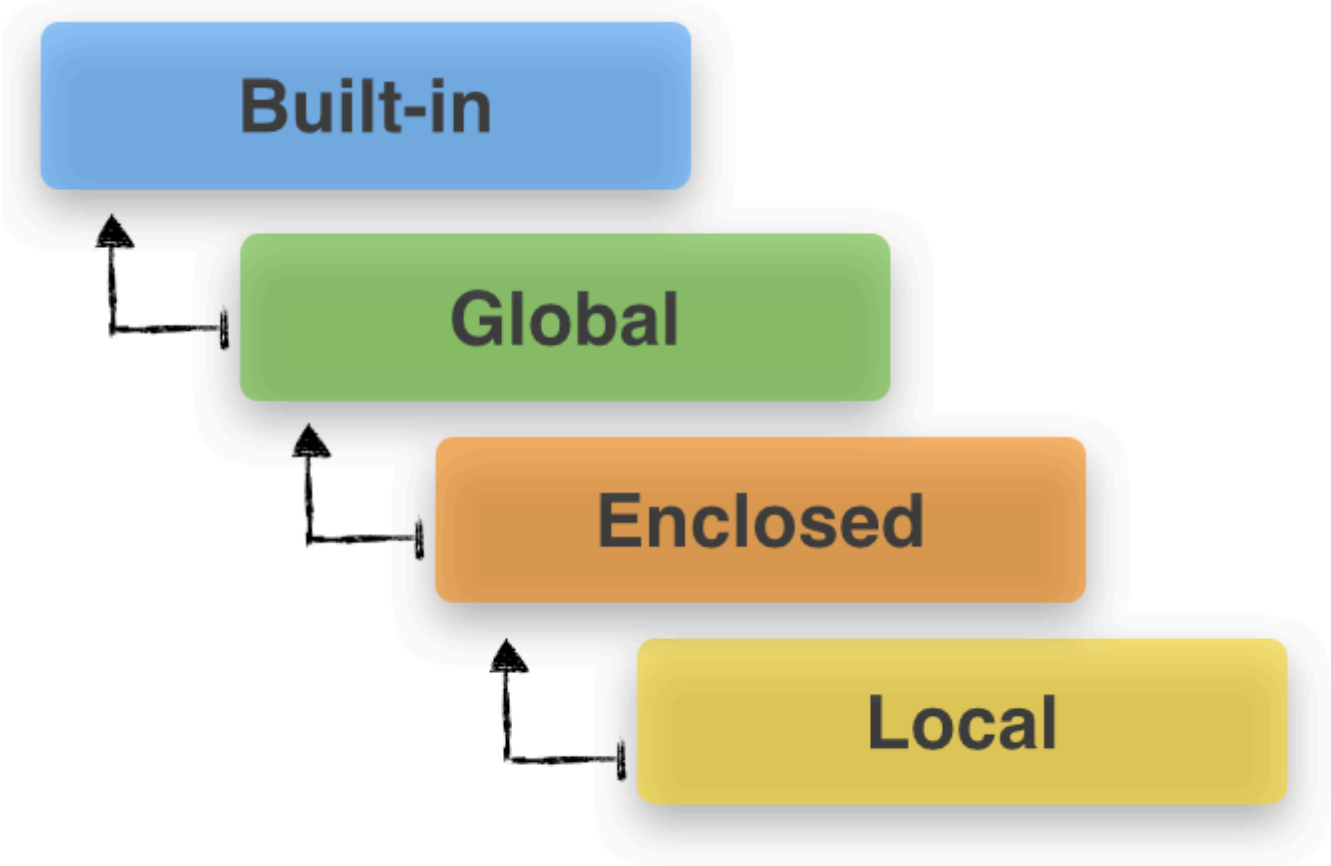
When an input is passed to a function, what does the function actually get?

- Internal variables get a *copy* of input values, with the exception of mutable objects

Local variables only exist within the function in which they are defined

- The variables cease to exist when the function ends
- The *scope* of a variable is the part(s) of code where that variable name binding is valid (i.e. where it exists)

Variable Scope (Python)



Variable Scope: Example I

```
i = 1

def foo():
    i = 5
    print(i, 'in foo()')

print(i, '=global')

foo()
```

Output?

1=global
5 in foo()

Variable Scope: Example II

```
a_var = 'global value'

def a_func():
    global a_var
    a_var = 'local value'
    print(a_var, '[ a_var inside
a_func() ]')

print(a_var, '[ a_var outside a_func() ]')
a_func()
print(a_var, '[ a_var outside a_func() ]')
```

Output?

global value [a_var outside a_func()]
local value [a_var inside a_func()]
local value [a_var outside a_func()]

Recursion

re·cur·sion
/riˈkərZHən/

noun MATHEMATICS LINGUISTICS

the repeated application of a recursive procedure or definition.

- a recursive definition.
  plural noun: **recursions**

re·cur·sive
/riˈkərsiv/

adjective

characterized by recurrence or repetition, in particular.

- MATHEMATICS LINGUISTICS
  relating to or involving the repeated application of a rule, definition, or procedure to successive results.
- COMPUTING
  relating to or involving a program or routine of which a part requires the application of the whole, so that its explicit interpretation requires in general many successive executions.

Recursive function calls itself, directly or indirectly

Reminder: Iteration

```
<initialization statements>
for <variables> in <sequence expression>:
  <body statements>

<rest of the program>
```

```
<initialization statements>
while <predicate expression>:
   <body statements>

<rest of the program>
```

```
[ <expr with loop var> for <loop var> in <sequence expr > ]
```

Iteration vs Recursion

For loop:

```
def sum(n):
    s=0
    for i in range(0,n+1):
        s=s+i
    return s
```

Iteration vs Recursion

While loop:

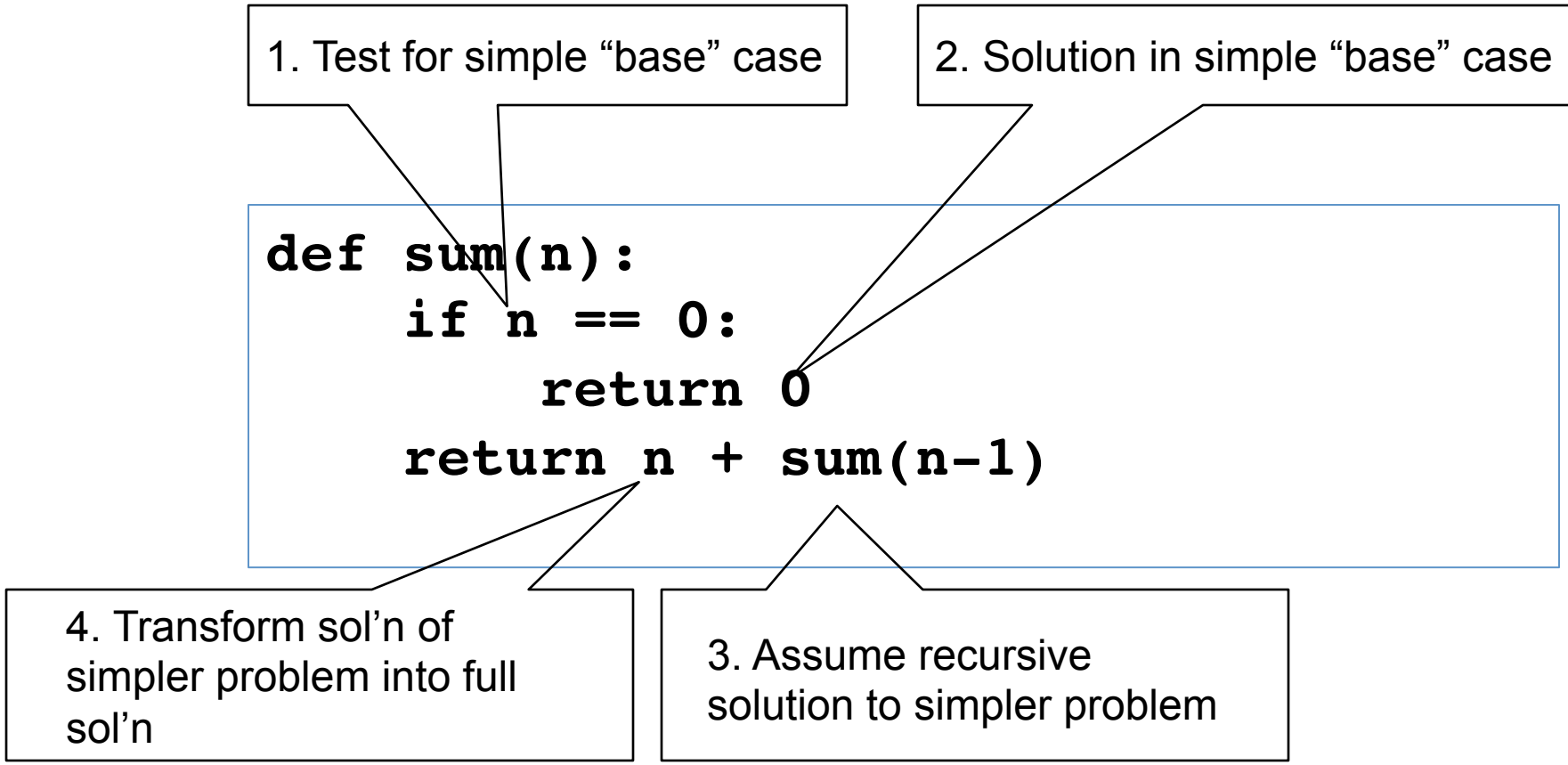
```
def sum(n):
    s=0
    i=0
    while i<n:
        i=i+1
        s=s+i
    return s
```

Iteration vs Recursion

Recursion:

```
def sum(n):
    if n==0:
        return 0
    return n+sum(n-1)
```

Recursion: Pattern



- **Linear recursion**

Why does it work?

```
sum(3)

# sum(3) => 3 + sum(2)
#                => 3 + sum(2) + sum(1)
#                => 3 + sum(2) + sum(1) + sum(0)
#                => 3 + sum(2) + sum(1) + 0
#                => 3 + sum(2) + 1
#                => 3 + 3
#                => 6
```

How does it work?

- Each recursive call gets its own local variables
  - Just like any other function call
- Computes its result (possibly using additional calls)
  - Just like any other function call
- Returns its result and returns control to its caller
  - Just like any other function call
- The function that is called happens to be itself
  - Called on a simpler problem
  - Eventually bottoms out on the simple base case

- Reason about correctness "by mathematical induction"
  - Solve a base case
  - Assuming a solution to a smaller problem, extend it

Local variables

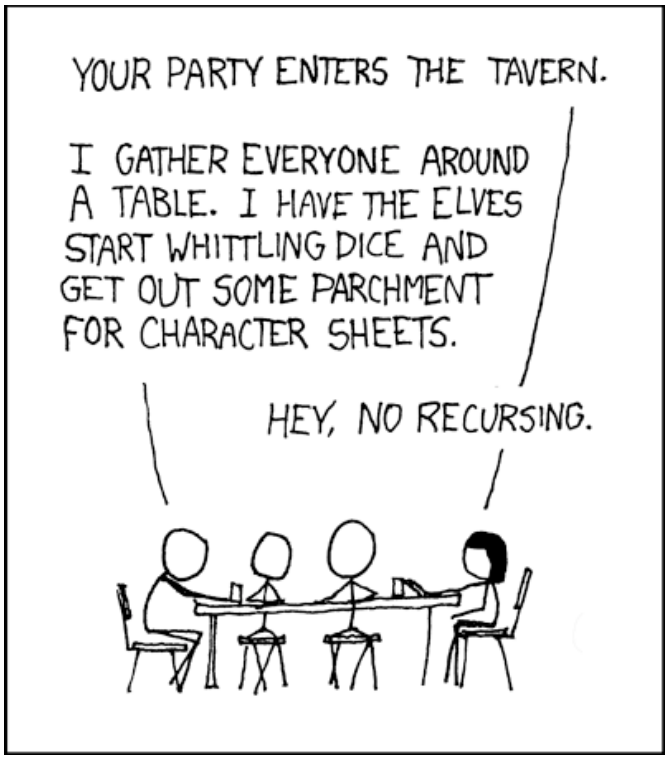
```
def sum(n):
    if n==0:
        return 0
    return n+sum(n-1)
```

Each call has its own "frame" of local variables

Sanity Check…

- **Recursion is ■ Iteration (i.e., loops)**

  a) more powerful than
  b) just as powerful as
  c) less powerful than



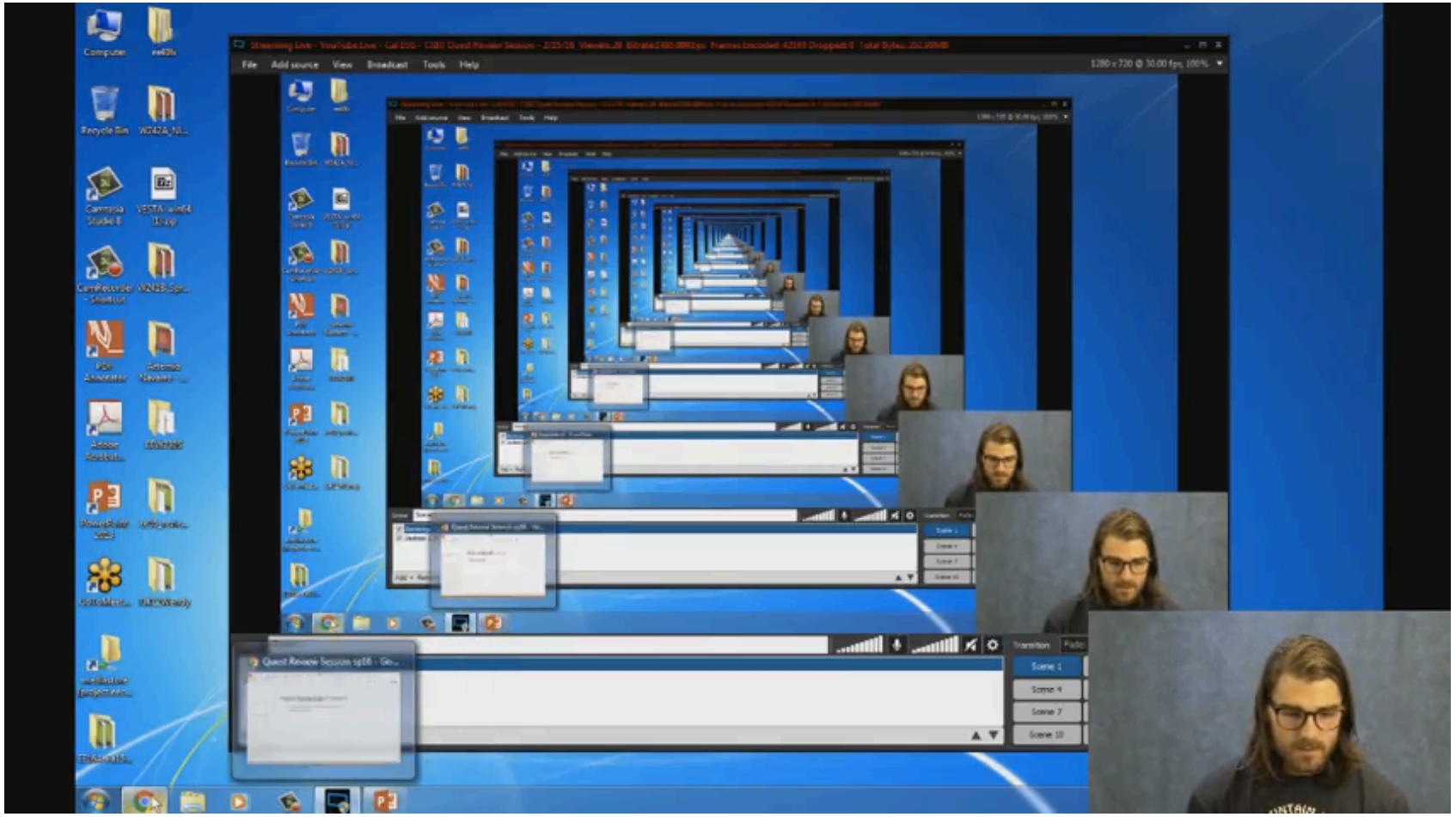
Why Recursion?

- "After Abstraction, Recursion is probably the 2nd biggest idea in this course"
- "It's tremendously useful when the problem is self-similar"
- "It's no more powerful than iteration, but often leads to more concise & better code"
- "It's more 'mathematical'"
- "It embodies the beauty and joy of computing"
- …

Why Recursion? More Reason

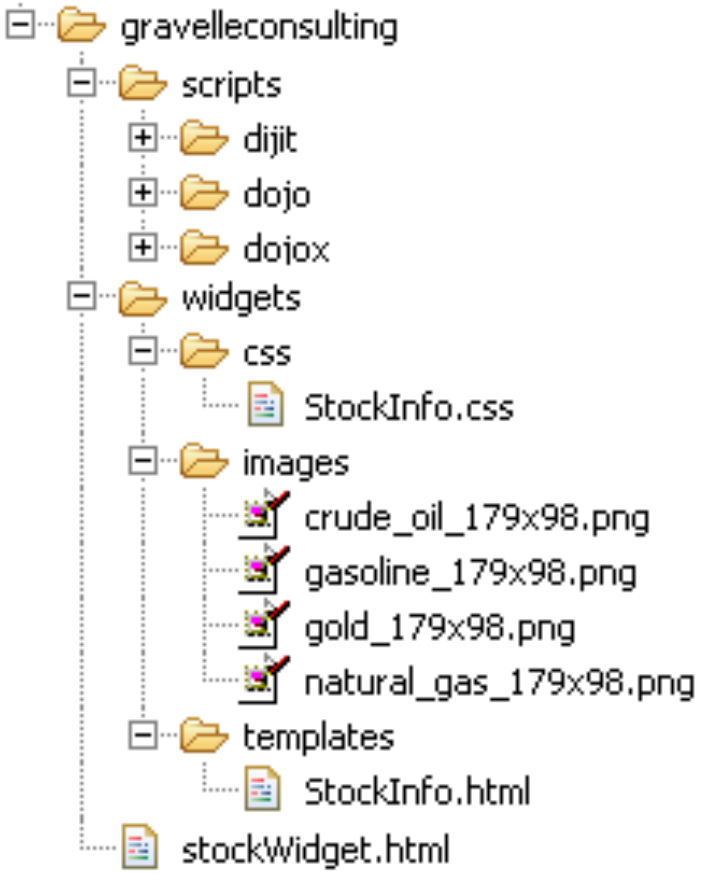
- Recursive structures exist (sometimes hidden) in nature and therefore in data!
- It's mentally and sometimes computationally more efficient to process recursive structures using recursion.

Recursion (unwanted)



Example I

List all items on your hard disk



- **Files**
- **Folders contain**
  - **Files**
  - **Folders**

Recursion!

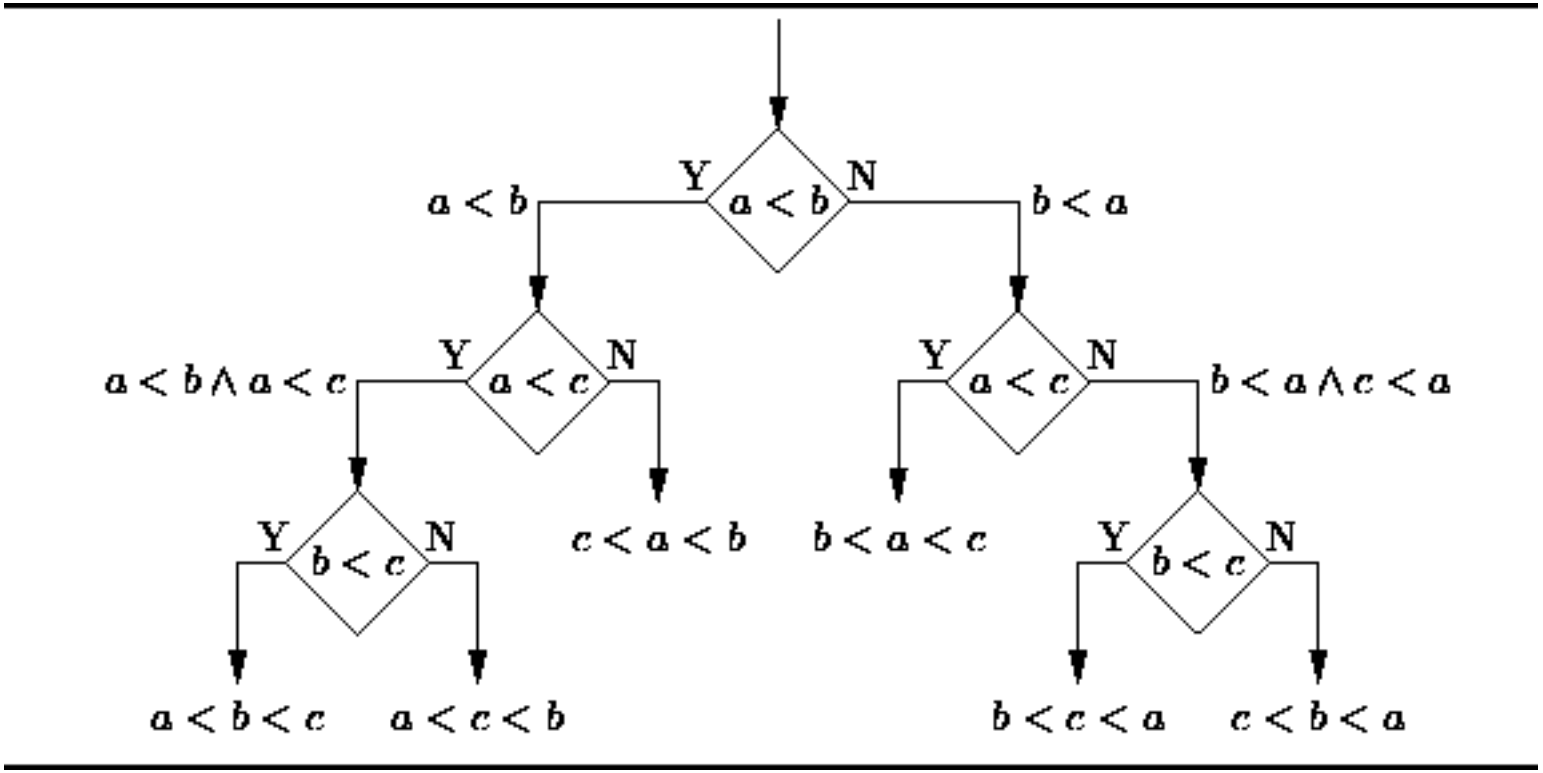
List Files in Python

```
def listfiles(directory):
    content = [os.path.join(directory, x) for x in os.listdir(directory)]

    dirs = sorted([x for x in content if os.path.isdir(x)])
    files = sorted([x for x in content if os.path.isfile(x)])

    for d in dirs:
        print d
        listfiles(d)

    for f in files:
        print f
```

Iterative version about twice as much code and much harder to think about.

Example II

Sort the numbers in a list.



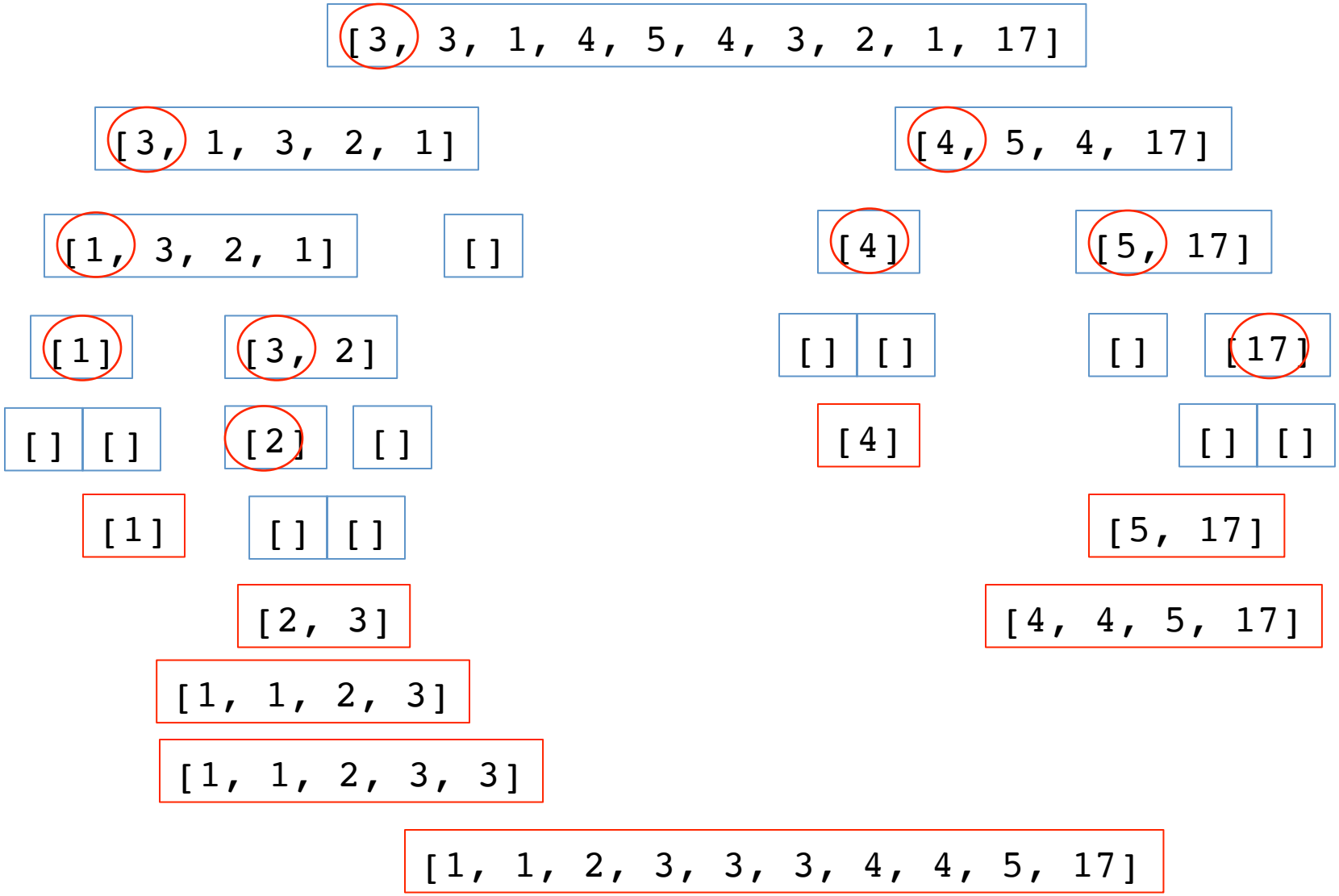
Hidden recursive structure: Decision tree!

Tree Recursion makes Sorting Efficient

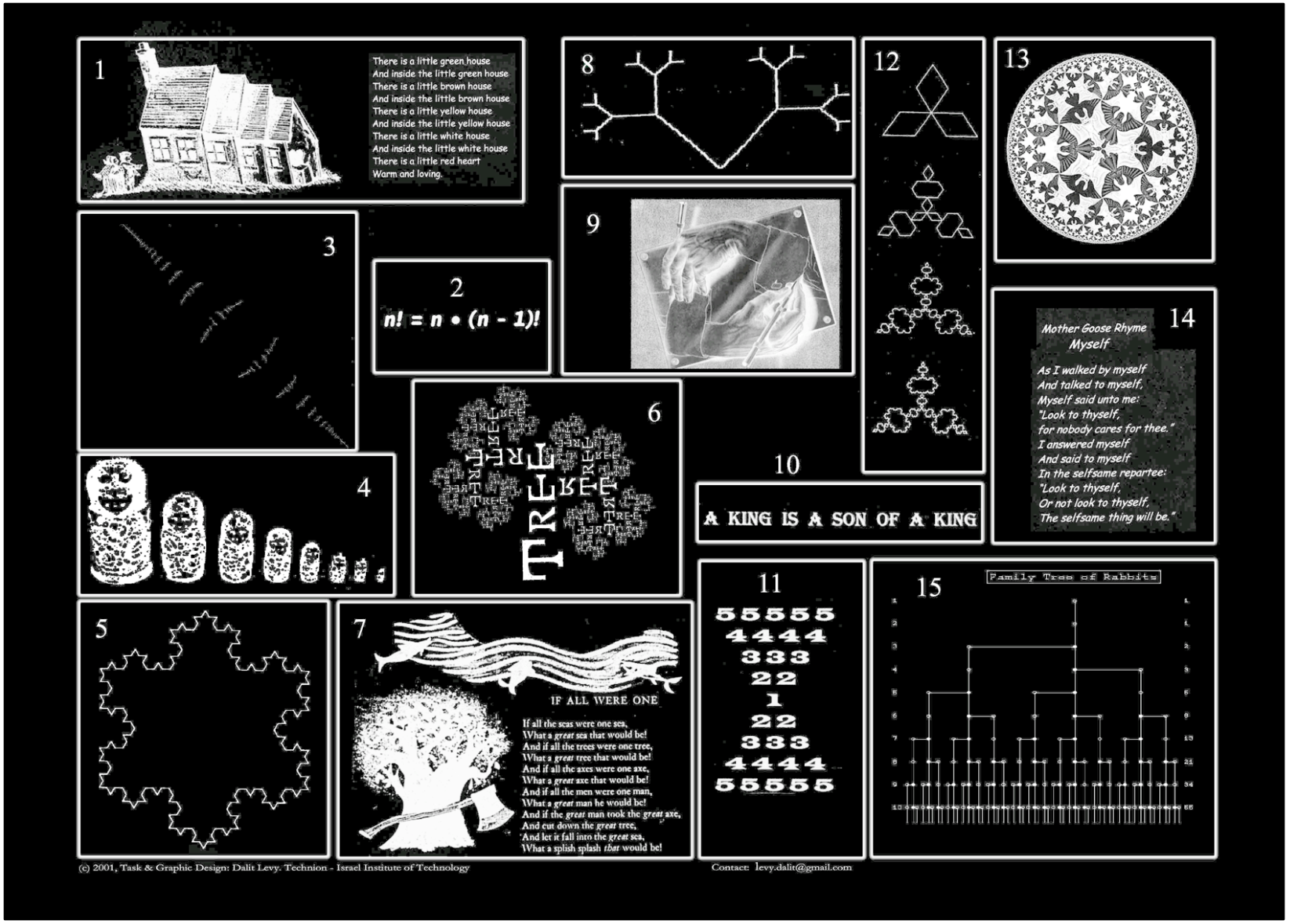
Break the problem into multiple smaller sub-problems, and solve them recursively

```
def split(x, s):
    return [i for i in s if i <= x], [i for i in s if i > x]

def qsort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""
    if not s:
        return []
    else:
        pivot = first(s)
        lessor, more = split(pivot, rest(s))
        return qsort(lessor) + [pivot] + qsort(more)

>>> qsort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```

QuickSort Example



Questions?


1
There is a little green house
And inside the little green house
There is a little brown house
And inside the little brown house
There is a little yellow house
And inside the little yellow house
There is a little white house
And inside the little white house
There is a little red heart
Warm and loving.
2
n! = n • (n - 1)!
3
4
5
6
TREE
7
IF ALL WERE ONE
If all the seas were one sea,
What a great sea that would be!
And if all the trees were one tree,
What a great tree that would be!
And if all the axes were one axe,
What a great axe that would be!
And if all the men were one man,
What a great man he would be!
And if the great man took the great axe,
And cut down the great tree,
And let it fall into the great sea,
What a splish splash that would be!
8
9
10
A KING IS A SON OF A KING
11
55555
4444
333
22
1
22
333
4444
55555
12
13
14
Mother Goose Rhyme
Myself
As I walked by myself
And talked to myself,
Myself said unto me:
"Look to thyself,
for nobody cares for thee."
I answered myself
And said to myself
In the selfsame repartee:
"Look to thyself,
Or not look to thyself,
The selfsame thing will be."
15
Family Tree of Rabbits
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