

Recursion

Recursion is when a function calls itself to solve a smaller version of the same problem. Instead of tackling a complex task all at once, recursion breaks it down into simpler steps until it reaches a **base case**.

Many students find this topic challenging. Everything gets easier with practice. Please help each other learn.

Q1: Swipe

Implement `swipe`, which prints the digits of argument `n`, one per line, **first backward then forward**. The left-most digit is printed only once. **Do not use** `while` or `for` or `str`. (Use recursion, of course!)

```
def swipe(n):  
    """Print the digits of n, one per line, first backward then forward.  
  
    >>> swipe(2837)  
    7  
    3  
    8  
    2  
    8  
    3  
    7  
    ""  
  
    if n < 10:  
        print(n)  
    else:  
        """ YOUR CODE HERE """
```

First `print` the first line of the output, then make a recursive call, then `print` the last line of the output.

Q2: Skip Factorial

Define the base case for the `skip_factorial` function, which returns the product of **every other positive** integer, starting with `n`.

```
def skip_factorial(n):  
    """Return the product of positive integers n * (n - 2) * (n - 4) * ...  
  
    >>> skip_factorial(5) # 5 * 3 * 1  
    15  
    >>> skip_factorial(8) # 8 * 6 * 4 * 2  
    384  
    """  
    if ___:  
        return ___  
    else:  
        return ___
```

If `n` is even, then the base case will be 2. If `n` is odd, then the base case will be 1. Try to write a condition that handles both possibilities.

Q3: Recursive Hailstone

Recall the `hailstone` function from [Homework 2](#). First, pick a positive integer `n` as the start. If `n` is even, divide it by 2. If `n` is odd, multiply it by 3 and add 1. Repeat this process until `n` is 1. Complete this recursive version of `hailstone` that prints out the values of the sequence and returns the number of steps.

```
def hailstone(n):
    """Print out the hailstone sequence starting at n,
    and return the number of elements in the sequence.
    >>> a = hailstone(10)
    10
    5
    16
    8
    4
    2
    1
    >>> a
    7
    >>> b = hailstone(1)
    1
    >>> b
    1
    """
    print(n)
    if n % 2 == 0:
        return even(n)
    else:
        return odd(n)

def even(n):
    return ----

def odd(n):
    """*** YOUR CODE HERE ***"
```

An even number is never a base case, so `even` always makes a recursive call to `hailstone` and returns one more than the length of the rest of the hailstone sequence.

An odd number might be 1 (the base case) or greater than one (the recursive case). Only the recursive case should call `hailstone`.

Extra Questions

The questions below are **optional but recommended** if you would like some extra practice.

Q4: Is Prime

Implement `is_prime` that takes an integer `n` greater than 1. It returns `True` if `n` is a prime number and `False` otherwise. Try following the approach below, but implement it recursively without using a `while` (or `for`) statement.

You will need to define another “helper” function (a function that exists just to help implement this one). Does it matter whether you define it within `is_prime` or as a separate function in the global frame? Try to define it to take as few arguments as possible.

```
def is_prime(n):
    """Returns True if n is a prime number and False otherwise.
    >>> is_prime(2)
    True
    >>> is_prime(16)
    False
    >>> is_prime(521)
    True
    """
    "*** YOUR CODE HERE ***"
```

Define an inner function that checks whether some integer between `i` and `n` evenly divides `n`. Then you can call it starting with `i=2`:

```
def is_prime(n):
    def f(i):
        if i == n:
            return ____
        elif ____:
            return ____
        else:
            return f(____)
    return f(2)
```

Q5: Function Repeater

Define a function `make_fn_repeater` which takes in a one-argument function `f` and an integer `x`. It should return another function which takes in one argument, another integer. This function returns the result of applying `f` to `x` this number of times.

Make sure to use recursion in your solution.

```
def make_func_repeater(f, x):
    """
    >>> increment_repeater = make_func_repeater(lambda x: x + 1, 1)
    >>> increment_repeater(2) #same as f(f(x))
    3
    >>> increment_repeater(5)
    6
    """
    def repeat(____):
        if ____:
            return ____
        else:
            return ____
    return ____
```