

## Linked Lists

A linked list is a `Link` object or `Link.empty`.

You can mutate a `Link` object `s` in two ways: - Change the first element with `s.first = ...` - Change the rest of the elements with `s.rest = ...`.

You can make a new `Link` object by calling `Link`: - `Link(4)` makes a linked list of length 1 containing 4. - `Link(4, s)` makes a linked list that starts with 4 followed by the elements of linked list `s`.

```
class Link:
    """A linked list is either a Link object or Link.empty

    >>> s = Link(3, Link(4, Link(5)))
    >>> s.rest
    Link(4, Link(5))
    >>> s.rest.rest.rest is Link.empty
    True
    >>> s.rest.first * 2
    8
    >>> print(s)
    (3 4 5)
    """
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest_repr = ', ' + repr(self.rest)
        else:
            rest_repr = ''
        return 'Link(' + repr(self.first) + rest_repr + ')'

    def __str__(self):
        string = '('
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + ')'
```

**Q1: Sum Two Ways**

Implement both `sum_rec` and `sum_iter`. Each one takes a linked list of numbers `s` and a non-negative integer `k` and returns the sum of the first `k` elements of `s`. If there are fewer than `k` elements in `s`, all of them are summed. If `k` is 0 or `s` is empty, the sum is 0.

Use recursion to implement `sum_rec`. Don't use recursion to implement `sum_iter`; use a `while` loop instead.

To get started on the recursive implementation, consider the example `a = Link(1, Link(6, Link(8)))`, and the call `sum_rec(a, 2)`. Write down the recursive call to `sum_rec` that would help compute `sum_rec(a, 2)`. Then, write down what that recursive call should return. Discuss how this return value is useful in computing the return value of `sum_rec(a, 2)`.

```
def sum_rec(s, k):
    """Return the sum of the first k elements in s.

    >>> a = Link(1, Link(6, Link(8)))
    >>> sum_rec(a, 2)
    7
    >>> sum_rec(a, 5)
    15
    >>> sum_rec(Link.empty, 1)
    0
    """
    # Use a recursive call to sum_rec; don't call sum_iter
    """*** YOUR CODE HERE ***"""

def sum_iter(s, k):
    """Return the sum of the first k elements in s.

    >>> a = Link(1, Link(6, Link(8)))
    >>> sum_iter(a, 2)
    7
    >>> sum_iter(a, 5)
    15
    >>> sum_iter(Link.empty, 1)
    0
    """
    # Don't call sum_rec or sum_iter
    """*** YOUR CODE HERE ***"""
```

Add `s.first` to the sum of the first `k-1` elements in `s.rest`. Your base case condition should include `s is Link.empty` so that you're checking whether `s` is empty before ever evaluating `s.first` or `s.rest`.

Introduce a new name, such as `total`, then repeatedly (in a `while` loop) add `s.first` to `total`, set `s = s.rest` to advance through the linked list, and reduce `k` by one.

**Discussion time:** When adding up numbers, the intermediate sums depend on the order.  $(1 + 3) + 5$  and  $1 + (3 + 5)$  both equal 9, but the first one makes 4 along the way while the second makes 8 along the way. For the same linked list `s` and length `k`, will `sum_rec` and `sum_iter` both make the same intermediate sums along the way?

**Q2: Duplicate Link**

Write a function `duplicate_link` that takes in a linked list `s` and a `value`. `duplicate_link` will mutate `s` such that if there is a linked list node that has a `first` equal to `value`, that node will be duplicated. Note that you should be **mutating the original linked list** `s`; you will need to create new `Links`, but you should not be returning a new linked list.

**Note:** In order to insert a link into a linked list, you need to modify the `.rest` of certain links. We encourage you to draw out a doctest to visualize!

```
def duplicate_link(s: Link, val: int) -> None:
    """Mutates s so that each element equal to val is followed by another val.

    >>> x = Link(5, Link(4, Link(5)))
    >>> duplicate_link(x, 5)
    >>> x
    Link(5, Link(5, Link(4, Link(5, Link(5)))))
    >>> y = Link(2, Link(4, Link(6, Link(8))))
    >>> duplicate_link(y, 10)
    >>> y
    Link(2, Link(4, Link(6, Link(8))))
    >>> z = Link(1, Link(2, Link(2, Link(3))))
    >>> duplicate_link(z, 2) # ensures that back to back links with val are both
    duplicated
    >>> z
    Link(1, Link(2, Link(2, Link(2, Link(2, Link(2, Link(3)))))))
    """
    "*** YOUR CODE HERE ***"
```