Trees, Efficiency

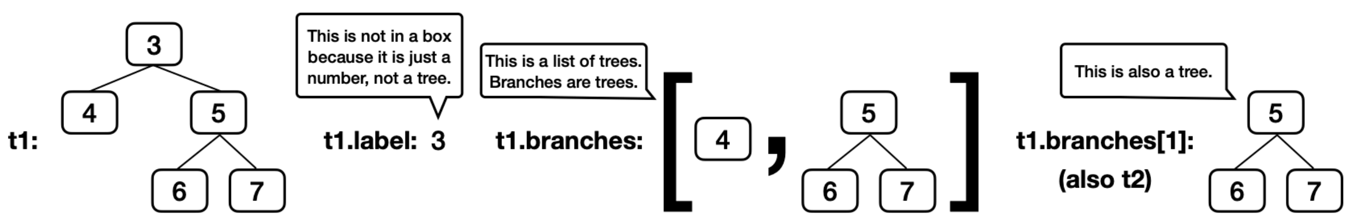
Discussion 9: November 3, 2025

Trees

For a `Tree` instance `t`: - Its root label can be any value, and `t.label` evaluates to it. - Its branches are all `Tree` instances, and `t.branches` evaluates to a list of branches, which is a list of `Tree` instances. - It is called a leaf if it has no branches, and `t.is_leaf()` returns whether `t` is a leaf. - A new `Tree` with the same root label and branches can be constructed with `Tree(t.label, t.branches)`.

Here's an example tree `t1`, for which its branch `t1.branches[1]` is `t2`.

```
t2 = Tree(5, [Tree(6), Tree(7)])
t1 = Tree(3, [Tree(4), t2])
```



Example Tree

A path is a sequence of nodes in which each is the parent of the next.

You don't need to know how the `Tree` class is implemented in order to use it correctly, but here is the implementation from lecture.

```
class Tree:
    """A tree is a label and a list of branches."""
    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)

    def is_leaf(self):
        return not self.branches

    # The rest of the class just determines how trees are displayed.

    def __repr__(self):
        if self.branches:
            branch_str = ', ' + repr(self.branches)
        else:
            branch_str = ''
        return 'Tree({0}{1})'.format(repr(self.label), branch_str)

    def __str__(self):
        return '\n'.join(self.indented())

    def indented(self):
        lines = []
        for b in self.branches:
            for line in b.indented():
                lines.append('  ' + line)
        return [str(self.label)] + lines
```

Q1: Min Tree

What value is bound to `result`?

```
get_label = lambda t: t.label
result = min(max([t1, t2], key=get_label).branches, key=get_label).label
```

Solution

6: `max([t1, t2], key=get_label)` evaluates to the `t2` tree because its label 5 is larger than `t1`'s label 3. Among `t2`'s branches (which are leaves), the left one labeled 6 has a smaller label.

Here's a quick refresher on how `key` functions work with `max` and `min`,

`max(s, key=f)` returns the item `x` in `s` for which `f(x)` is largest.

```
>>> s = [-3, -5, -4, -1, -2]
>>> max(s)
-1
>>> max(s, key=abs)
-5
>>> max([abs(x) for x in s])
5
```

Therefore, `max([t1, t2], key=get_label)` returns the tree with the largest label, in this case `t2`.

Q2: Has Path

Implement `has_path`, which takes a `Tree` instance `t` and a list `p`. It returns whether there is a path from the root of `t` with labels `p`. For example, `t1` has a path from its root with labels `[3, 5, 6]` but not `[3, 4, 6]` or `[5, 6]`.

Important: Before trying to implement this function, discuss these questions from lecture about the recursive call of a tree processing function: - What recursive calls will you make? - What type of values do they return? - What do the possible return values mean? - How can you use those return values to complete your implementation?

If you get stuck, you can view our answers to these questions by clicking the hint button below.

What recursive calls will you make?

As you usual, you will call `has_path` on each branch `b`. You'll make this call after comparing `p[0]` to `t.label`, and so the second argument to `has_path` will be the rest of `p`: `has_path(b, p[1:])`.

What type of values do they return?

`has_path` always returns a `bool` value: `True` or `False`.

What do the possible return values mean?

If `has_path(b, p[1:])` returns `True`, then there is a path through branch `b` for which `p[1:]` are the node labels.

How can you use those return values to complete your implementation?

If you have already checked that `t.label` is equal to `p[0]`, then a `True` return value means there is a path through `t` with labels `p` using that branch `b`. A `False` value means there is no path through that branch, but there might be path through a different branch.

```python
def has_path(t, p):
    """Return whether Tree t has a path from the root with labels p.

    >>> t2 = Tree(5, [Tree(6), Tree(7)])
    >>> t1 = Tree(3, [Tree(4), t2])
    >>> has_path(t1, [5, 6])        # This path is not from the root of t1
    False
    >>> has_path(t2, [5, 6])        # This path is from the root of t2
    True
    >>> has_path(t1, [3, 5])        # This path does not go to a leaf, but that's ok
    True
    >>> has_path(t1, [3, 5, 6])     # This path goes to a leaf
    True
    >>> has_path(t1, [3, 4, 5, 6])  # There is no path with these labels
    False
    """
    if p == [t.label]:
        return True
    elif t.label != p[0]:
        return False
    else:
        for b in t.branches:
            if has_path(b, p[1:]):
                return True
        return False
```

Efficiency

Tips for finding the order of growth of a function's runtime:

- If the function is recursive, determine the number of recursive calls and the runtime of each recursive call.
- If the function is iterative, determine the number of inner loops and the runtime of each loop.
- Ignore coefficients. A function that performs `n` operations and a function that performs `100 * n` operations are both linear.
- Choose the largest order of growth. If the first part of a function has a linear runtime and the second part has a quadratic runtime, the overall function has a quadratic runtime.
- In this course, we only consider constant, logarithmic, linear, quadratic, and exponential runtimes.

Q3: The First Order...of Growth

What is the efficiency of `rey`?

```
def rey(finn):
    poe = 0
    while finn >= 2:
        poe += finn
        finn = finn / 2
    return
```

Choose one of:

- Constant
- Logarithmic
- Linear
- Quadratic
- Exponential
- None of these

Solution: Logarithmic, because our while loop iterates at most log(`finn`) times, due to `finn` being halved in every iteration. This is commonly known as $\Theta(\log(\texttt{finn}))$ runtime. Another way of looking at this if you duplicate the input, we only add a single iteration to the time, which also indicates logarithmic.

What is the efficiency of `mod_7`?

```
def mod_7(n):
    if n % 7 == 0:
        return 0
    else:
        return 1 + mod_7(n - 1)
```

Choose one of:

- Constant
- Logarithmic
- Linear
- Quadratic
- Exponential
- None of these

Solution: Constant, since in the worst case scenario our function `mod_7` will require 6 recursive calls to reach the base case. Consider the worst case where we have an input `n` such that our first call to `mod_7` evaluates `n % 7` as `6`. Each recursive call will decrement `n` by `1`, allowing us to eventually reach the base case of returning `0` in 6 recursive calls (`n` will range from 0 to 6). Since the growth of the computation is independent of the input, we say this is constant, which is commonly known as a $\Theta(1)$ runtime.