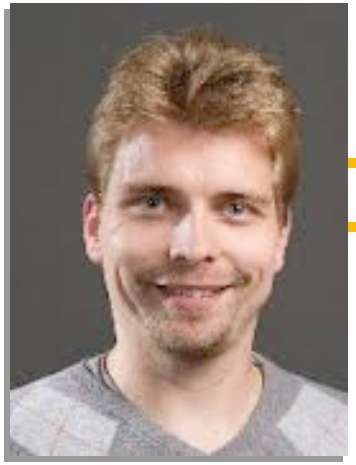

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Computational Structures in Data Science

Lecture 6: Environment Diagrams, Recursion Review, Midterm Review

March 4, 2019

http://inst.eecs.berkeley.edu/~cs88

Recursion Key concepts – by example

1. Test for simple “base” case

2. Solution in simple “base” case

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

3. Assume recusive solution to simpler problem

4. Transform soln of simpler problem into full soln

How does it work?

- Each recursive call gets its own local variables
  - Just like any other function call
- Computes its result (possibly using additional calls)
  - Just like any other function call
- Returns its result and returns control to its caller
  - Just like any other function call
- The function that is called happens to be itself
  - Called on a simpler problem
  - Eventually bottoms out on the simple base case

- Reason about correctness “by induction”
  - Solve a base case
  - Assuming a solution to a smaller problem, extend it

Local variables

```
def sum_of_squares(n):
    n_squared = n**2
    if n < 1:
        return 0
    else:
        return n_squared + sum_of_squares(n-1)
```

- **Each call has its own "frame" of local variables**
- **What about globals?**
- **Let's see the environment diagrams**

https://goo.gl/CiFaUJ

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Edit code

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Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Edit code

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pythontutor.com

Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n | 3

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Edit code

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Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Edit code

Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

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Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

Environments Example

Python 3.3

```
1  def sum_of_squares(n):
2      n_squared = n**2
3      if n == 1:
4          return 1
5      else:
6          return n_squared + sum_of_squares(n-1)
7
8  sum_of_squares(3)
```

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Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

Python 3.3

```
1  def sum_of_squares(n):
2      n_squared = n**2
3      if n == 1:
4          return 1
5      else:
6          return n_squared + sum_of_squares(n-1)
7
8  sum_of_squares(3)
```

Edit code

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that has just executed
line to execute

Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

f3: sum_of_squares [parent=Global]
n 1

Environments Example

Python 3.3

```
1  def sum_of_squares(n):
2      n_squared = n**2
3      if n == 1:
4          return 1
5      else:
6          return n_squared + sum_of_squares(n-1)
7
8  sum_of_squares(3)
```

Edit code

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that has just executed
t line to execute

Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

f3: sum_of_squares [parent=Global]
n 1
n_squared 1

Python 3.3

```
1  def sum_of_squares(n):
2      n_squared = n**2
3      if n == 1:
4          return 1
5      else:
6          return n_squared + sum_of_squares(n-1)
7
8  sum_of_squares(3)
```

Edit code

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that has just executed
t line to execute

Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

f3: sum_of_squares [parent=Global]
n 1
n_squared 1

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Edit code

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e that has just executed
xt line to execute

Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

f3: sum_of_squares [parent=Global]
n 1
n_squared 1
Return value 1

Environments Example

Python 3.3

```
1  def sum_of_squares(n):
2      n_squared = n**2
3      if n == 1:
4          return 1
5      else:
6          return n_squared + sum_of_squares(n-1)
7
8  sum_of_squares(3)
```

Edit code

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e that has just executed
xt line to execute

Frames Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4
Return value 5

f3: sum_of_squares [parent=Global]
n 1
n_squared 1
Return value 1

Environments Example

Python 3.3

```
1  def sum_of_squares(n):
2      n_squared = n**2
3      if n == 1:
4          return 1
5      else:
6          return n_squared + sum_of_squares(n-1)
7
8  sum_of_squares(3)
```

Edit code

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e that has just executed
xt line to execute

Frames Objects

Global frame

sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]

n	3
n_squared	9
Return value	14

f2: sum_of_squares [parent=Global]

n	2
n_squared	4
Return value	5

f3: sum_of_squares [parent=Global]

n	1
n_squared	1
Return value	1

How much ???

- **Time is required to compute `sum_of_squares(n)`?**
  - **Recursively?**
  - **Iteratively ?**
- **Space is required to compute `sum_of_squares(n)`?**
  - **Recursively?**
  - **Iteratively ?**

Linear
proportional to n
c*n* for some c

- **Count the frames…**
- **Recursive is linear, iterative is constant!**

Tail Recursion

- All the work happens on the way down the recursion
- On the way back up, just return

```
def sum_up_squares(i, n, accum):
    """Sum the squares from i to n in incr. order"""
    if i > n:
        Base Case
    else:
        Tail Recursive Case


>>> sum_up_squares(1,3,0)
14
```

Tree Recursion

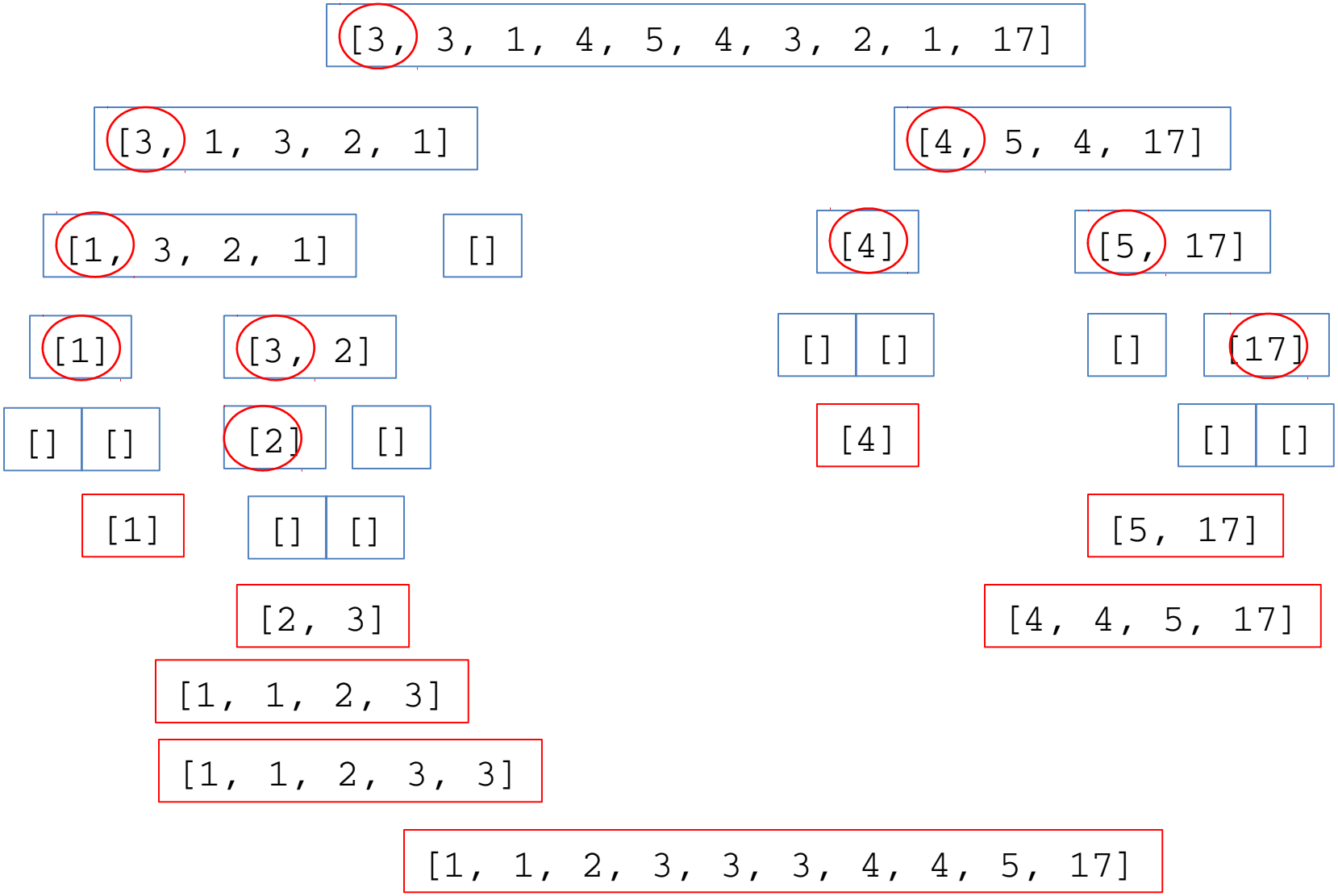
- **Break the problem into multiple smaller sub-problems, and Solve them recursively**

```
def split(x, s):
    return [i for i in s if i <= x], [i for i in s if i > x]

def qsort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""
    if not s:
        return []
    else:
        pivot = first(s)
        lessor, more = split(pivot, rest(s))
        return qsort(lessor) + [pivot] + qsort(more)

>>> qsort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```

QuickSort Example



Tree Recursion with HOF

```
def qsort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""

    if not s:
        return []
    else:
        pivot = first(s)
        lessor, more = split_fun(leq_maker(pivot), rest(s))
        return qsort(lessor) + [pivot] + qsort(more)

>>> qsort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```

Computational Concepts Toolbox

- Data type: values, literals, operations,
  - e.g., int, float, string
- Expressions, Call expression
- Variables
- Assignment Statement
- Sequences: tuple, list
  - indexing
- Data structures
- Tuple assignment
- Call Expressions
- Function Definition Statement
- Conditional Statement

- Iteration:
  - data-driven (list comprehension)
  - control-driven (for statement)
  - while statement
- Higher Order Functions
  - Functions as Values
  - Functions with functions as argument
  - Assignment of function values
- Recursion
- Environment Diagrams

Answers for the Wandering Mind

The computer choses a random element *x* of the list generated by `range(0,n)`. What is the <u>smallest amount</u> of iteration/recursion steps the best algorithm needs to guess *x*?

$\log_2 n$

How would the algorithm look like?

Guess the binary digits of x starting with the highest significant digit. This is, ask questions of the form
"smaller than 2^{n-1}?" (yes => 0…),
"smaller than 2^{n-2}?" (no => 0 1…),
"smaller than $2^{n-2}+2^{n-3}$?", …

This method is also called: binary search

Quantum physics: Allow less than $\log_2 n$ guesses