

HIGHER ORDER FUNCTIONS 3

COMPUTER SCIENCE 88

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1 Higher Order Functions

A **higher order function** (HOF) is a function that manipulates other functions by taking in functions as arguments, returning a function, or both.

1.1 Functions as Arguments

One way a higher order function can exploit other functions is by taking functions as input. Consider this higher order function called `negate`.

```
def negate(f, x):  
    return -f(x)
```

`negate` takes in a function `f` and a number `x`. It doesn't care what exactly `f` does, as long as `f` takes in a number and returns a number. Its job is simple: call `f` on `x` and return the negation of that value.

1.2 Questions

- Here are some possible functions that can be passed through as `f`.

```
def square(n):  
    return n * n
```

```
def double(n):  
    return 2 * n
```

What will the following Python statements output?

```
>>> negate(square, 5)
```

```
>>> negate(double, -19)
```

```
>>> negate(double, negate(square, -4))
```

2. Implement a function `keep_ints`, which takes in a function `cond` and a number `n`, and only prints a number from 1 to `n` if calling `cond` on that number returns `True`:

```
def keep_ints(cond, n):
```

```
    """Print out all integers 1..i..n where cond(i) is true
```

```
>>> def is_even(x):
```

```
...     # Even numbers have remainder 0 when divided by 2.
```

```
...     return x % 2 == 0
```

```
>>> keep_ints(is_even, 5)
```

```
2
```

```
4
```

```
"""
```

1.3 Functions as Return Values

Often, we will need to write a function that returns another function. One way to do this is to define a function inside of a function:

```
def outer(x):  
    def inner(y):  
        ...  
    return inner
```

The return value of `outer` is the function `inner`. This is a case of a function returning a function. In this example, `inner` is defined inside of `outer`. Although this is a common pattern, we can also define `inner` outside of `outer` and still use the same return statement.

```
def inner(y):  
    ...  
def outer(x):  
    return inner
```

1.4 Questions

1. Use this definition of `outer` to fill in what Python would print when the following lines are evaluated.

```
def outer(n):  
    def inner(m):  
        return n - m  
    return inner  
>>> outer(61)  
  
>>> f = outer(10)  
>>> f(4)  
  
>>> outer(5)(4)
```

2. Implement a function `keep_ints` like before, but now it takes in a number `n` and returns a function that has one parameter `cond`. The returned function prints out all numbers from `1..i..n` where calling `cond(i)` returns `True`.

```
def keep_ints(n):  
    """Returns a function which takes one parameter cond and  
    prints out all integers 1..i..n where calling cond(i)  
    returns True.  
  
    >>> def is_even(x):  
    ...     # Even numbers have remainder 0 when divided by 2.  
    ...     return x % 2 == 0  
    >>> keep_ints(5)(is_even)  
    2  
    4  
    """
```

2 Environment Diagrams

1. Draw the environment diagram for evaluating the following code

```
def f(x):  
    return y + x  
y = 10  
f(8)
```

2. Draw the environment diagram for evaluating the following code

```
def dessef(a, b):  
    c = a + b  
    b = b + 1  
  
b = 6  
dessef(b, 4)
```

3. Draw the environment diagram for evaluating the following code

```
def foo(x, y):  
    foo = bar  
    return foo(bar(x, x), y)
```

```
def bar(z, x):  
    return z + y
```

```
y = 5  
foo(1, 2)
```