

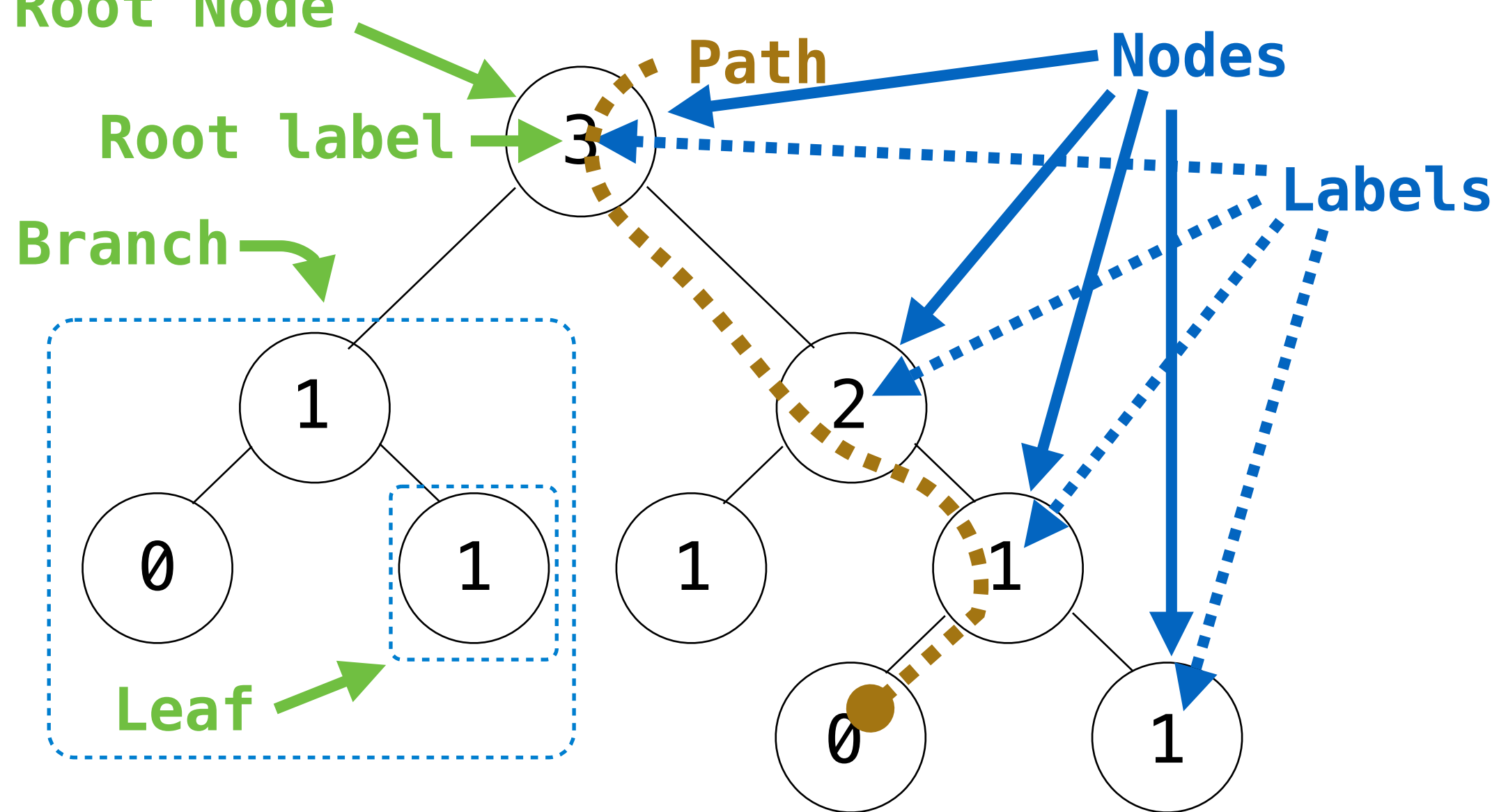
Root or Root Node

Recursive description:

- A **tree** has a root **label** and a list of **branches**
- Each branch is a **tree**
- A tree with zero branches is called a **leaf**

Relative description:

- Each location is a **node**
- Each **node** has a **label**
- One node can be the **parent/child** of another

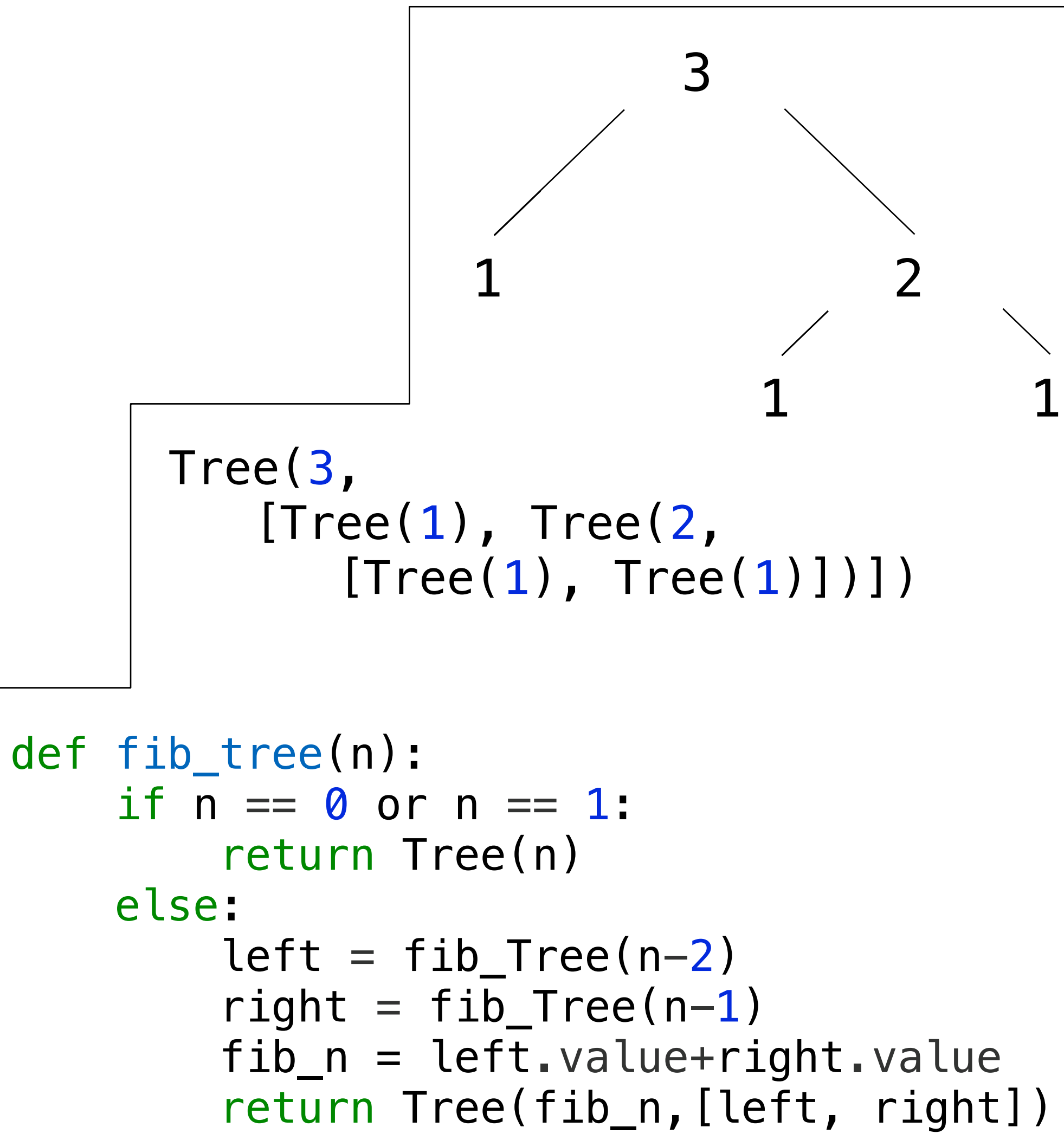


```
class Tree:
    def __init__(self, value, branches=[]):
        self.value = value
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
```

Built-in `isinstance` function: returns True if `branch` has a class that is or inherits from `Tree`

```
def is_leaf(self):
    return not self.branches
```

```
def leaves(tree):
    "The leaf values in a tree."
    if tree.is_leaf():
        return [tree.value]
    else:
        lst = []
        for b in tree.branches:
            lst.extend(leaves(b))
        return lst
```



Exceptions are raised with a `raise` statement.

```
raise <expr>
```

<expr> must evaluate to a subclass of `BaseException` or an

```
try:
    <try suite>
except <exception class> as <name>:
    <except suite>
```

```
>>> try:
    x = 1/0
except ZeroDivisionError as e:
    print('handling a', type(e))
    x = 0

handling a <class 'ZeroDivisionError'>
>>> x
0
```

The <try suite> is executed first.

If, during the course of executing the <try suite>, an exception is raised that is not handled otherwise, and

If the class of the exception inherits from <exception class>, then

The <except suite> is executed, with <name> bound to the exception.

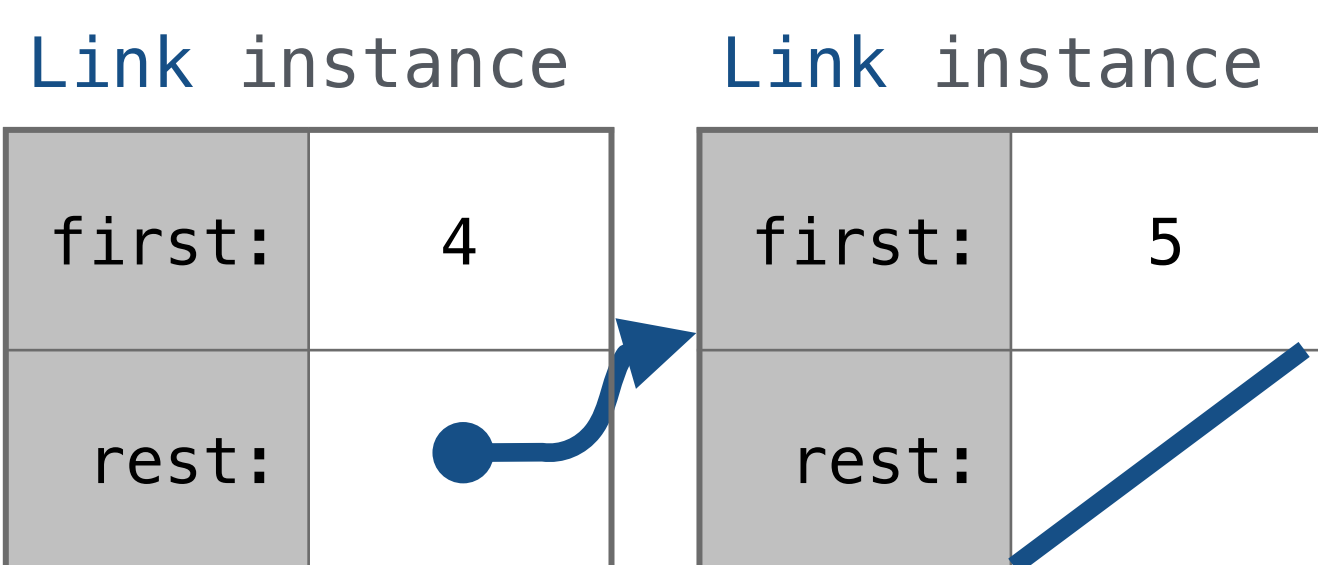
```
class Link:
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest = ',' + repr(self.rest)
        else:
            rest = ''
        return 'Link(' + repr(self.first) + rest + ')'

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + '>'
```

Some zero length sequence



```
>>> s = Link(4, Link(5))
>>> s
Link(4, Link(5))
>>> s.first
4
>>> s.rest
Link(5)
>>> print(s)
<4 5>
>>> print(s.rest)
<5>
>>> s.rest.rest is Link.empty
True
```

Anatomy of a recursive function:

- The **def statement header** is like any function
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**
- Recursive cases are evaluated **with recursive calls**

```
def sum_digits(n):
    "Sum the digits of positive integer n."
    if n < 10:
        return n
    else:
        all_but_last, last = n // 10, n % 10
        return sum_digits(all_but_last) + last
```

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Recursive decomposition: finding simpler instances of a problem.

- E.g., `count_partitions(6, 4)`
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - `count_partitions(2, 4)`
  - `count_partitions(6, 3)`
- Tree recursion often involves exploring different choices.

Python object system:

**Idea:** All bank accounts have a **balance** and an account **holder**; the **Account** class should add those attributes to each of its instances

A new instance is created by calling a class

```
>>> a = Account('Jim')
>>> a.holder
'Jim'
>>> a.balance
0
```

An account instance

balance: 0 holder: 'Jim'

When a class is called:

1. A new instance of that class is created:
2. The `__init__` method of the class is called with the new object as its first argument (named `self`), along with any additional arguments provided in the call expression.

```
class Account:
    def __init__(self, account_holder):
        self.balance = 0
        self.holder = account_holder
    def deposit(self, amount):
        self.balance = self.balance + amount
        return self.balance
    def withdraw(self, amount):
        if amount > self.balance:
            return 'Insufficient funds'
        self.balance = self.balance - amount
        return self.balance
```

`__init__` is called a constructor

`self` should always be bound to an instance of the `Account` class or a subclass of `Account`

Function call: all arguments within parentheses

```
>>> type(Account.deposit)
<class 'function'>
>>> type(a.deposit)
<class 'method'>
```

Method invocation: One object before the dot and other arguments within parentheses

```
>>> Account.deposit(a, 5)
10
>>> a.deposit(2)
12
```

Call expression

<expression> . <name>

The <expression> can be any valid Python expression.

The <name> must be a simple name.

Evaluates to the value of the attribute looked up by <name> in the object that is the value of the <expression>.

To evaluate a dot expression:

1. Evaluate the <expression> to the left of the dot, which yields the object of the dot expression
2. <name> is matched against the instance attributes of that object; if an attribute with that name exists, its value is returned
3. If not, <name> is looked up in the class, which yields a class attribute value
4. That value is returned unless it is a function, in which case a bound method is returned instead

Assignment statements with a dot expression on their left-hand side affect attributes for the object of that dot expression

- If the object is an instance, then assignment sets an instance attribute
- If the object is a class, then assignment sets a class attribute

Account class attributes

interest: ~~0.02~~ ~~0.04~~ 0.05  
(withdraw, deposit, \_\_init\_\_)

Instance attributes of jim\_account

balance: 0  
holder: 'Jim'  
interest: 0.08

Instance attributes of tom\_account

balance: 0  
holder: 'Tom'

```
>>> jim_account = Account('Jim')
>>> tom_account = Account('Tom')
>>> tom_account.interest
0.02
>>> jim_account.interest
0.02
>>> Account.interest = 0.04
>>> tom_account.interest
0.04
>>> jim_account.interest
0.04
```

```
>>> jim_account.interest = 0.08
>>> jim_account.interest
0.08
>>> tom_account.interest
0.04
>>> Account.interest = 0.05
>>> tom_account.interest
0.05
>>> jim_account.interest
0.08
```

```
class CheckingAccount(Account):
    """A bank account that charges for withdrawals."""
    withdraw_fee = 1
    interest = 0.01
    def withdraw(self, amount):
        return Account.withdraw(self, amount + self.withdraw_fee)
```

return super().withdraw(amount + self.withdraw\_fee)

To look up a name in a class:

1. If it names an attribute in the class, return the attribute value.
2. Otherwise, look up the name in the base class, if there is one.

```
>>> ch = CheckingAccount('Tom') # Calls Account.__init__
>>> ch.interest # Found in CheckingAccount
0.01
>>> ch.deposit(20) # Found in Account
20
>>> ch.withdraw(5) # Found in CheckingAccount
14
```





```
iter(iterable):
    Return an iterator
    over the elements of
    an iterable value
next(iterator):
    Return the next element

>>> s = [3, 4, 5]
>>> t = iter(s)
>>> next(t)
3
>>> next(t)
4

>>> d = {'one': 1, 'two': 2, 'three': 3}
>>> k = iter(d)
>>> next(k)
'one'
>>> next(k)
'two'
```

A *generator function* is a function that *yields* values instead of *returning* them.

```
>>> def plus_minus(x):
...     yield x
...     yield -x

>>> t = plus_minus(3)
>>> next(t)
3
>>> next(t)
-3

def a_then_b(a, b):
    yield from a
    yield from b

>>> list(a_then_b([3, 4], [5, 6]))
[3, 4, 5, 6]
```

Efficiency

**Constant growth:**  $O(1)$   
E.g., accessing a value from a dictionary  
Increasing  $n$  doesn't affect time

**Logarithmic growth:**  $O(\log(n))$   
E.g., binary search  
Doubling  $n$  only increments time by a constant

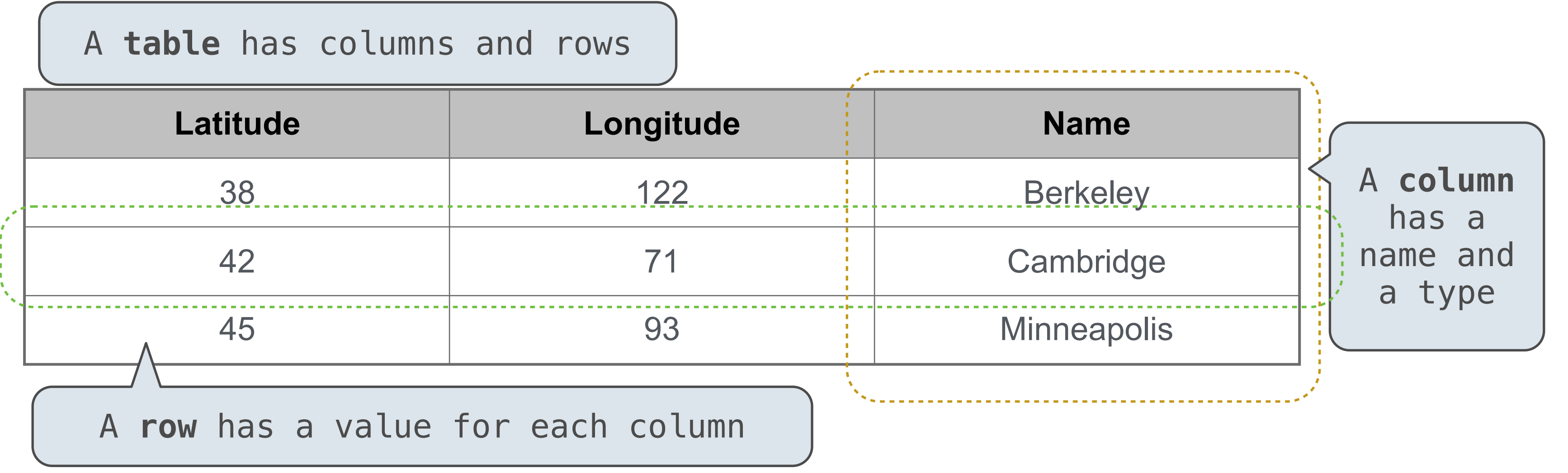
**Linear growth:**  $O(n)$   
E.g., iterating over a list of length  $n$   
Incrementing  $n$  increases time by a constant

**Quadratic growth:**  $O(n^2)$   
E.g., finding all pairs of a list of integers (double for loop)  
Incrementing  $n$  increases time by  $n$  times a constant

**Exponential growth:**  $O(b^n)$   
E.g., recursive fib  
Incrementing  $n$  multiplies time by a constant

```
def perms(lst):
    """
    Generates the permutations of lst one by one.
    """
    >>> perms = perms([1, 2, 3])
    >>> p = list(perms)
    >>> p.sort()
    >>> p
    [[1, 2, 3], [1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1]]

    if lst == []:
        yield []
    else:
        for perm in perms(lst[1:]):
            for i in range(len(lst)):
                yield perm[:i] + [lst[0]] + perm[i:]
```

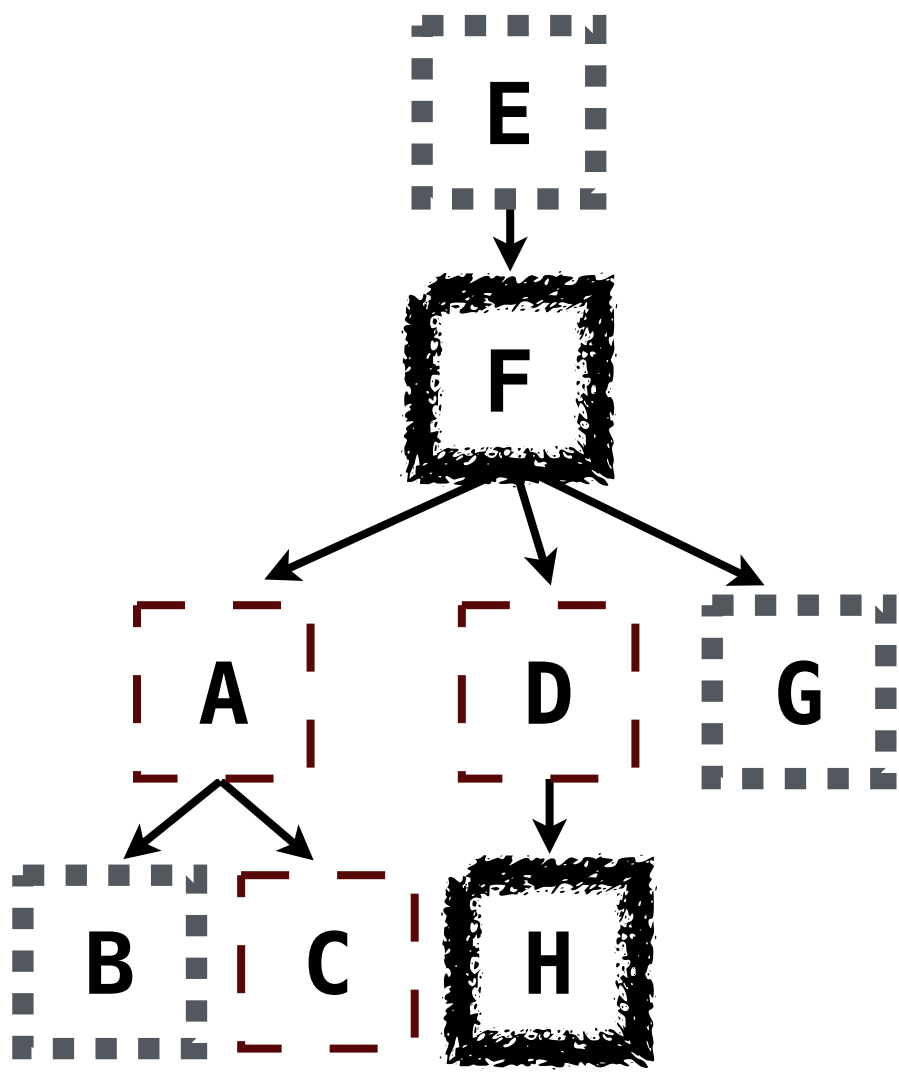


```
SELECT [expression] AS [name], [expression] AS [name], ... ;
SELECT [columns] FROM [table] WHERE [condition] ORDER BY [order];
```

```
CREATE TABLE parents AS
SELECT "abraham" AS parent, "barack" AS child
UNION
SELECT "abraham"      , "clinton"
UNION
SELECT "delano"       , "herbert"
UNION
```

```
CREATE TABLE dogs AS
SELECT "abraham" AS name, "long" AS fur UNION
SELECT "barack"    , "short"  UNION
SELECT "clinton"   , "long"   UNION
SELECT "delano"    , "long"   UNION
SELECT "eisenhower", "short"  UNION
SELECT "fillmore"  , "curly"  UNION
SELECT "grover"    , "short"  UNION
SELECT "herbert"   , "curly";
```

```
SELECT a.child AS first, b.child AS second
FROM parents AS a, parents AS b
WHERE a.parent = b.parent AND a.child <
```



First	Second
barack	clinton
abraham	delano
abraham	grover
delano	grover

The number of groups is the number of unique values of an expression

A *having* clause filters the set of groups that are aggregated

```
select weight/legs, count(*) from animals
      group by weight/legs
      having count(*)>1;
```

weight/legs	count(*)	kind	legs	weight
5	2	dog	4	20
2	2	cat	4	10
		ferret	4	10
		parrot	2	6
		penguin	2	10
		t-rex	2	12000