Tree Recursion

Announcements

Recursion Review

How to Know That a Recursive Implementation is Correct

Tracing: Diagram the whole computational process (only feasible for very small examples)

Induction: Check f(0), then check that f(n) is correct as long as f(n−1) **...** f(0) are.

Abstraction: Assume f is correct (on simpler examples), then use it to implement f.

Recursive Process

1: Divide – Break the problem down into smaller parts.

2: Invoke – Make the actual recursive call.

3. Combine – Use the result of the recursive call in your result.

```
def fact(n):
    """Compute n factorial.

    >>> fact(5)
    120
    >>> fact(0)
    1
    """
    if n == 0 or n == 1:
        return 1
    else:
        return fact(n-1) * n
```

Simple Problem: Palindrome

1: Divide – Break the problem down into smaller parts.

2: Invoke – Make the actual recursive call.

3. Combine – Use the result of the recursive call in your result.

```
all_but_first = lambda word: word[1:] # hello -> ello
all_but_last = lambda word: word[:-1] # hello -> hell

def palindrome(word):
    “""
    >>> palindrome('c88c')
    True    """
    if len(word) <= 1:
        return True
    elif word[0] == word[-1]:
        return _____________
    else:
        return False
```

Simple Problem: Palindrome

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all_but_first = lambda word: word[1:] # hello -> ello
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def palindrome(word):
    “""
    >>> palindrome(‘c88c')
    True    """
    if len(word) <= 1:
        return True
    elif word[0] == word[-1]:
        return palindrome(all_but_first(all_but_last(word)))
    else:
        return False
```

Tree Recursion

Tree Recursion

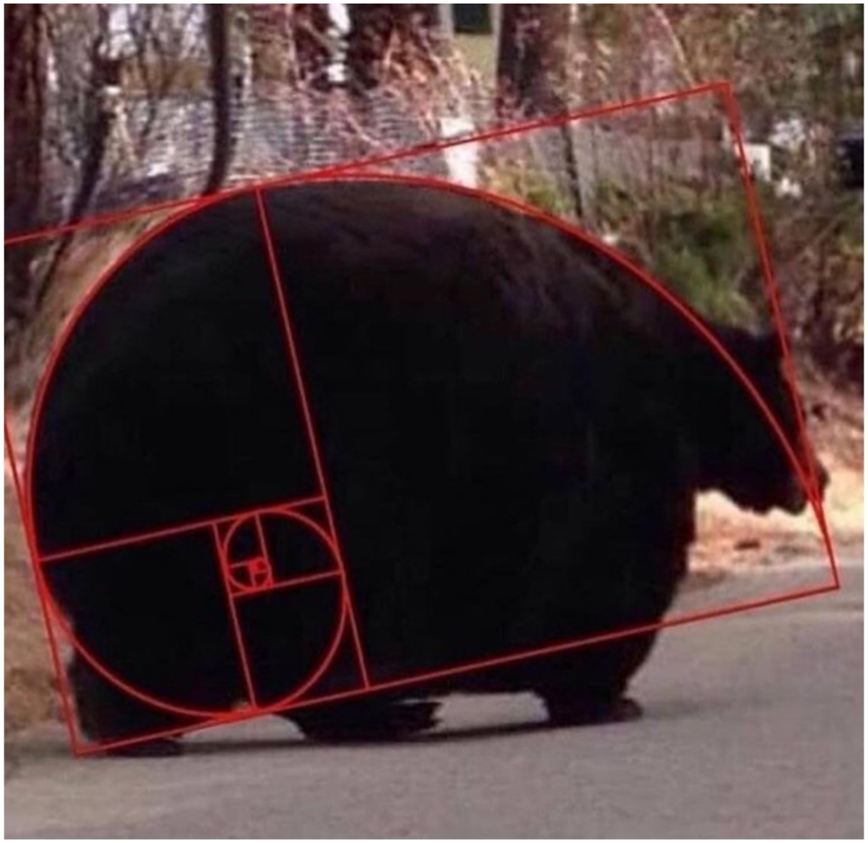
Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	9,227,465

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Go Bears!



Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

```
count_partitions(6, 4)
```

2 + 4 = 6

1 + 1 + 4 = 6

3 + 3 = 6

1 + 2 + 3 = 6

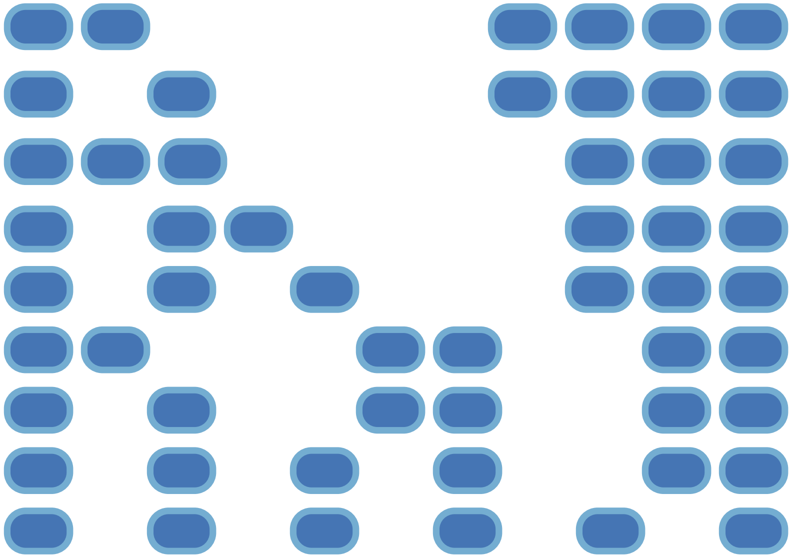
1 + 1 + 1 + 3 = 6

2 + 2 + 2 = 6

1 + 1 + 2 + 2 = 6

1 + 1 + 1 + 1 + 2 = 6

1 + 1 + 1 + 1 + 1 + 1 = 6

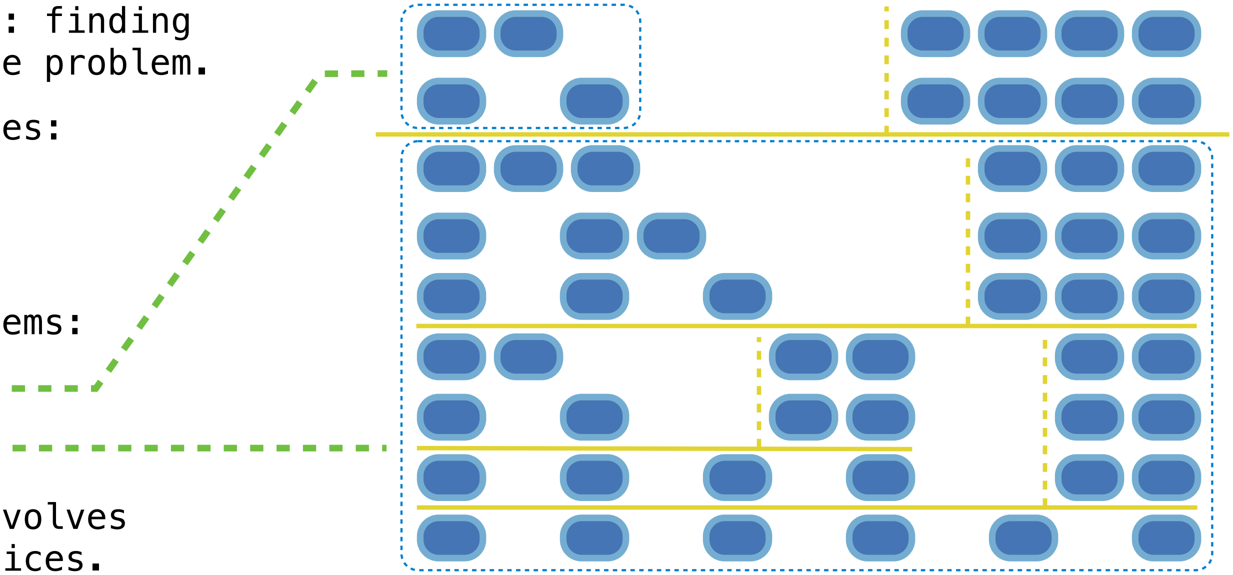


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in non-decreasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.



Counting Partitions

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  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

(Demo)

pythontutor.com/composingprograms.html#code=def%20count_partitions%28n,%20m%29%3A%0A%20%20%20%20if%20n%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%201%0A%20%20%20%20elif%20n%20<%200%3A%0A%20%20%20%20%20%20%20%20return%200%0A%20%20%20%20elif%20m%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%200%0A%20%20%20%20else%3A%0A%20%20%20%20%20%20%20%20with_m%20%3D%20count_partitions%28n-m,
%20m%29%20%0A%20%20%20%20%20%20%20%20without_m%20%3D%20count_partitions%28n,
%20m-1%29%0A%20%20%20%20%20%20%20%20return%20with_m%20%2B%20without_m%0A%20%20%20%20%20%20%20%20%0Aresult%20%3D%20count_partitions%285,%203%29%0A%0A#%201%20%2B%201%20%2B%201%20%2B%201%20%2B%201%20%3D%205%0A#%201%20%2B%201%20%2B%201%20%2B%202%20%2B%20%20%20%3D%205%0A#%201%20%2B%202%20%2B%202%20%2B%20%20%20%20%20%20%3D%205%0A#%201%20%2B%201%20%2B%203%20%2B%20%20%20%20%20%20%20%3D%205%0A#
20%2B%203%20%2B%20%20%20%20%20%20%20%20%20%20%20%20%20%3D%205&mode=display&origin=composingprograms.js&cumulative=false&py=3&rawInputLstJSON=[]&curInstr=0

Spring 2023 Midterm 2 Question 5

Definition. When parking vehicles in a row, a motorcycle takes up 1 parking spot and a car takes up 2 adjacent parking spots. A string of length n can represent n adjacent parking spots using % for a motorcycle, <> for a car, and . for an empty spot.

For example: '.%%.<><>' (Thanks to the Berkeley Math Circle for introducing this question.)

Implement **count_park**, which returns the number of ways that vehicles can be parked in n adjacent parking spots for positive integer n. Some or all spots can be empty.

```
def count_park(n):
    """Count the ways to park cars and motorcycles in n adjacent spots.
    >>> count_park(1)  # '.' or '%'
    2
    >>> count_park(2)  # '..', '.%', '%.', '%%', or '<>'
    5
    >>> count_park(4)  # some examples: '<><>', '.%%.', '%<>%', '%.<>'
    29
    """
    if n < 0:
        return ______0______
    elif n == 0:
        return ______1______
    else:
        return count_park(n-2) + count_park(n-1) + count_park(n-1)
```