

# Computational Structures in Data Science

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Efficiency  
& Run Time Analysis

Berkeley  
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# Learning Objectives

- Runtime Analysis:
  - How long will my program take to run?
  - Why can't we just use a clock?
  - How can we simplify understanding computation in an algorithm
- Enjoy this stuff? Take 61B!
- Find it challenging? Don't worry! It's a different way of thinking.

# Efficiency is all about trade-offs

- Running Code: Takes Time, Requires Memory
  - More efficient code takes less time or uses less memory
- Any computation we do, requires both time and "space" on our computer.
- Writing efficient code is not obvious
  - Sometimes it is even convoluted!
- But!
- We need a framework before we can optimize code
- Today, we're going to focus on the time component.

# Is this code fast?

- Most code doesn't *really* need to be fast! Computers, even your phones are already amazingly fast!
- Sometimes...it does matter!
  - Lots of data
  - Small hardware
  - Complex processes
- Slow code takes up battery power

# Beware!

"Premature Optimization is the root of all evil"

- Donald Knuth, Stanford CS Professor

There is **no use** in fast code if it is wrong!

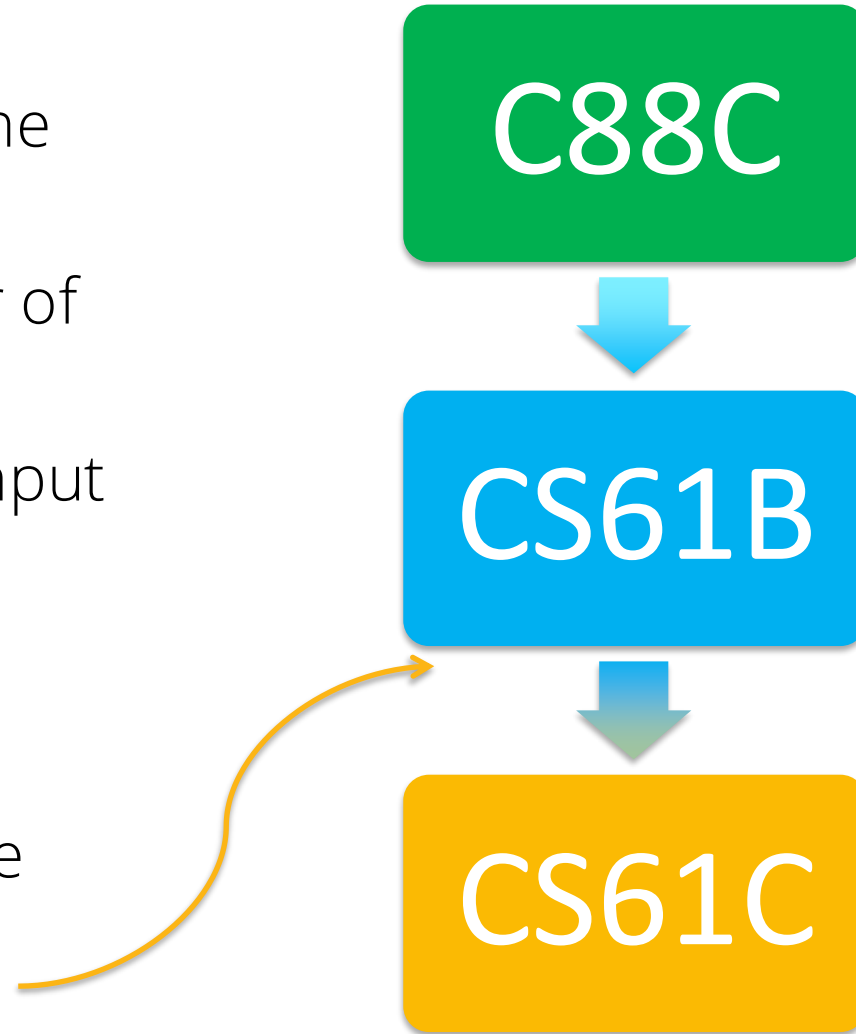
# Runtime analysis problem & solution

- Time w/stopwatch, but...
  - Different computers may have different runtimes. ☹️
  - Same computer may have different runtime on the same input. ☹️
  - Need to implement the algorithm first to run it. ☹️
- *Solution*: Count the number of “steps” involved, not time
  - Each operation = 1 step
    - $1 + 2$  is one step
    - `lst[5]` is one step
  - *When we say “runtime”, we’ll mean # of steps, not time!*



# Runtime: input size & efficiency

- Definition:
  - **Input size**: the # of things in the input.
  - e.g. length of a list, the number of iterations in a loop.
  - Running time as a function of input size
  - Measures **efficiency**
- Important!
  - In CS88 we won't care about the efficiency of your solutions!
  - ...in CS61B we will



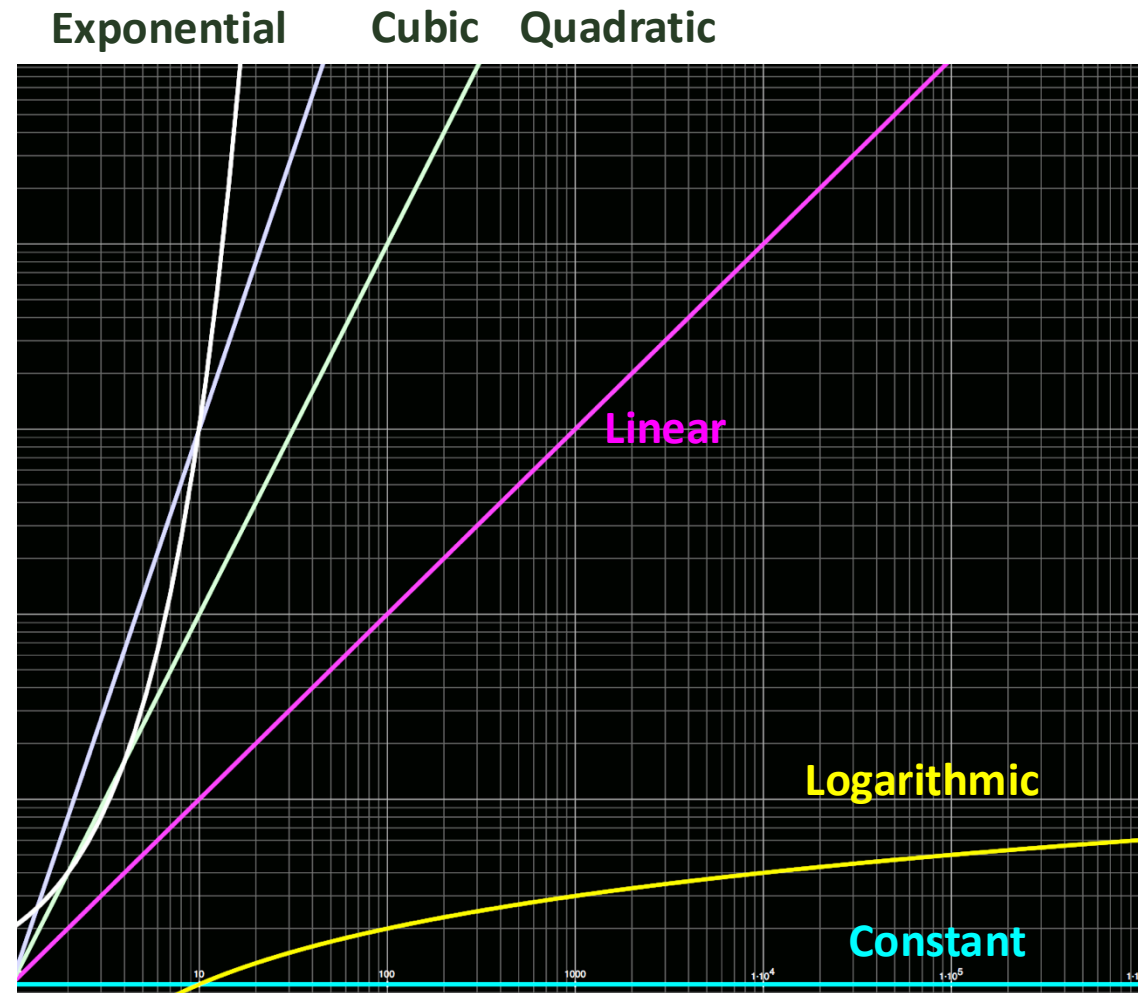
# Runtime analysis : worst or average case?

- Could use avg case:
  - Average running time over a vast # of inputs
- Instead: use worst case
  - Consider running time as input grows
- Why?
  - Nice to know most time we'd ever spend
  - Worst case happens often
  - The "average" can be similar to the worst
- Often called "Big O" for "order"
  - $O(1)$ ,  $O(n)$  ...



# Runtime analysis: Final abstraction

- Instead of an exact number of operations we'll use abstraction
  - Want order of growth, or dominant term
- In CS88 we'll consider
  - Constant  $O(1)$
  - Logarithmic  $O(\log n)$
  - Linear  $O(n)$
  - Quadratic  $O(n^2)$
  - Exponential  $O(2^n)$
- E.g.  $10n^2 + 4\log(n) + n$ 
  - ...is quadratic



Graph of order of growth curves  
on log-log plot

# Example: Finding a student (by ID)

- Input
  - Unsorted list of students L
  - Find student S
- Output
  - True if S is in L, else False
- Pseudocode Algorithm
  - Go through one by one, checking for match.
  - If match, true
  - If exhausted L and didn't find S, false



- Worst-case running time as function of the size of L?
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

# Computational Patterns

- If the number of steps to solve a problem is always the same → Constant time:  $O(1)$
- If the number of steps increases similarly for each larger input → Linear Time:  $O(n)$ 
  - Most commonly: **for each item**
- If the number of steps increases by some a factor of the input → Quadratic Time:  $O(n^2)$ 
  - Most commonly: Nested for Loops
- Two harder cases:
  - Logarithmic Time:  $O(\log n)$ 
    - We can double our input with only one more level of work
    - Dividing data in “half” (or thirds, etc)
  - Exponential Time:  $O(2^n)$

# Example: Finding a student (by ID)

- Input
  - Sorted list of students L
  - Find student S
- Output : same
- Pseudocode Algorithm
  - Start in middle
  - If match, report true
  - If exhausted, throw away half of L and check again in the middle of remaining part of L
  - If nobody left, report false



- Worst-case running time as function of the size of L?
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

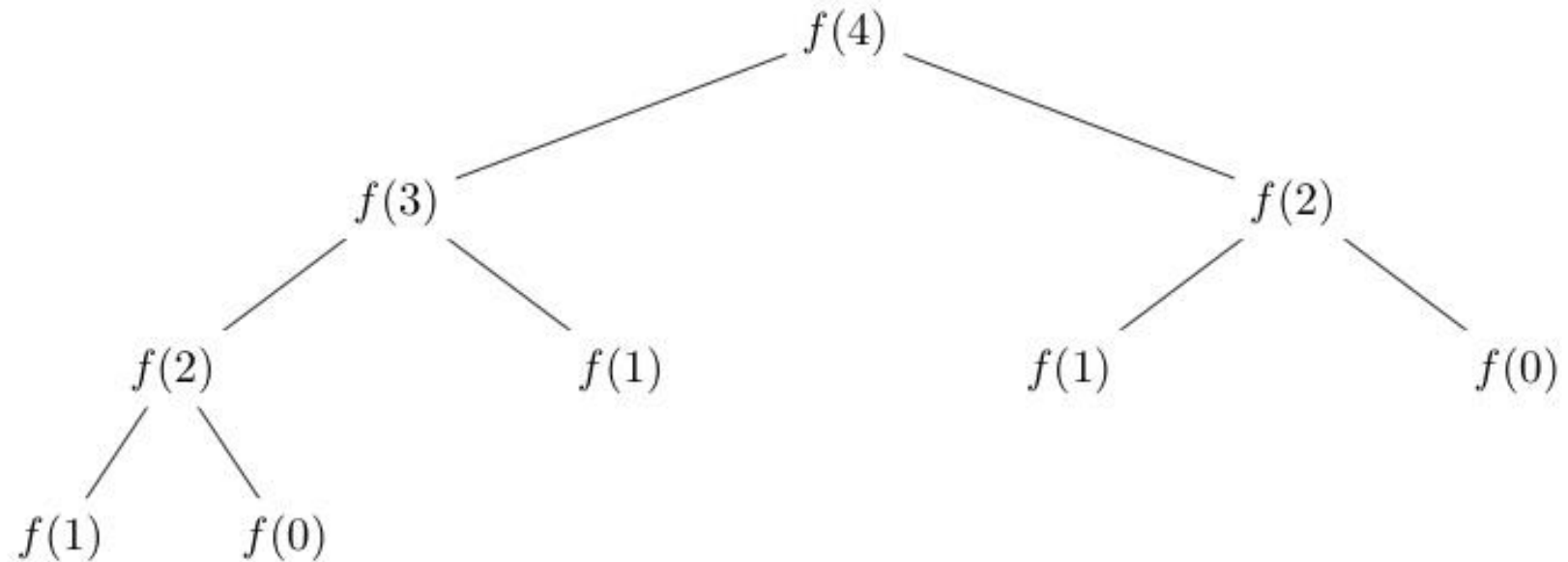
# Comparing Fibonacci

```
def iter_fib(n):  
    x, y = 0, 1  
    for _ in range(n):  
        x, y = y, x+y  
    return x
```

```
def fib(n): # Recursive  
    if n < 2:  
        return n  
    return fib(n - 1) + fib(n - 2)
```

# Tree Recursion

- Fib(4) → 9 Calls
- Fib(5) → 16 Calls
- Fib(6) → 26 Calls
- Fib(7) → 43 Calls
- Fib(20) →



# Why?

- Notice there was all this duplication in the tree?
- What is the exact order of growth?
  - It's exponential.
  - phi to the N ( $\phi^n$ ), where phi is the golden ratio.

<b>N</b>	<b>Operations</b>
1	1
2	3
3	5
4	9
7	41
8	67
20	21891

# Efficiency of Linked Lists vs Lists

- Linked Lists generally use less memory.
  - But this can make it slower to *compute* data.
- Linked Lists:
  - Once you've found an item, inserting / removing is easy,  $O(1)$
  - Finding anything other than the first/last item is  $O(n)$
- "Regular" Lists:
  - Inserting / Removing items, other than the last is  $O(n)$  – due to internal copying
  - Finding any random item is  $O(1)$ .
- What if you need to iterate over all items in order?
  - $O(n)$  in both cases

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More Examples



# Read and Analyze this code

```
def my_pow(base, exp):  
    result = 1  
    for _ in range(exp):  
        result *= base  
    return result
```

```
my_pow(2, 10) == 1024
```

- Worst-case running time as function of the **exp**?
  1. Constant
  2. Logarithmic
  3. **Linear**
  4. Quadratic
  5. Exponential

# Read and Analyze this code

```
def my_pow(base, exp):  
    result = 1  
    while exp > 0:  
        if exp % 2 == 1:  
            result *= base  
        base *= base  
        exp //= 2  
    return result
```

- Worst-case running time as function of the exp?
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

# Read and Analyze this code

```
def mystery_b(n):  
    if n == 0:  
        return [[]]  
    rest = mystery_b(n - 1)  
    return rest + [s + [n] for s in rest]
```

- Worst-case running time as function of  $n$ ?
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

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Improving Efficiency

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# Reminder!

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# Learning Objectives

- Learn how to cache the results to save time.
- "memoization" is a specific version to avoid repeated calculations

# Example

- Use a dictionary to cache results.
- This is called *memoization*

```
fib_results = {}
def memo_fib(n): # Look up values in our dictionary.
    global fib_results
    if n in fib_results:
        print(f'found {n} -> {fib_results[n]}')
        return fib_results[n]
    if n < 2:
        fib_results[n] = n
        return n
    result = memo_fib(n - 1) + memo_fib(n - 2)
    fib_results[n] = result
    return result
```

# A Better Approach

- [Python's functools module](#) has a `cache` function
- Uses a technique called decorators that we don't cover.
  - Decorators are really just a "shortcut" for higher order functions.
  - e.g. `cache_fib = cache(fib)` is a similar approach to the function below, but less commonly used.

```
from functools import cache
```

```
@cache
```

```
def cache_fib(n): # Recursive
```

```
    if n < 2:
```

```
        return n
```

```
    return cache_fib(n - 1) + cache_fib(n - 2)
```

# What next?

- Understanding *algorithmic complexity* helps us know whether something is possible to solve.
- Gives us a formal reason for understanding why a program might be slow
- This is only the beginning:
  - We've only talked about time complexity, but there is *space complexity*.
  - In other words: How much memory does my program require?
  - Often you can trade time for space and vice-versa
  - Tools like "caching" and "memorization" do this.
- If you think this is cool take CS61B!