

Computational Structures in Data Science

Lecture 5 Higher Order Functions

Week 2, Summer 2024. 6/25 (Tues)

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Announcements

Exam dates

Midterm: Wednesday July 17th, 3PM – 5PM PST

Final: Wednesday August 7th, 3PM – 5PM PST

Exams will be **administered online**, and **proctored via Zoom**. You may need to present your ID (eg student CalID card, or any ID with your name + photo) during the Zoom call to proctors.

Important: for those that can't make the above exam times, we will have **alternate exam times**. Stay tuned for details here!

Announcements

- Do watch [Ed](#) for announcements
 - Please remember to pick the best category when asking questions
 - Use the Python code option

Announcements

- Lab 2 released today (Due: June 29)
- Homework 2 released today (Due: June 29)
- Remember to do your lecture self checks!
- Reminder: you must submit all assignments to [Gradescope](#).
- okpy is only a convenience tool. Its backups don't count as submitting to Gradescope.

Today's Overview

- List comprehensions
- Higher order functions
- Environment Diagrams

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List Comprehensions

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Learning Objectives

- List comprehensions let us build lists "inline".
- List comprehensions are an *expression that returns a list*.
- We can easily "filter" the list using a conditional expression, i.e. **if**

Data-driven iteration: List Comprehensions

- describe an expression to perform on each item in a sequence
- let the data dictate the control
- In some ways, nothing more than a concise for loop.
- Always returns a list!

```
[ <expr with loop var> for <loop var> in <sequence expr > ]
```

```
[ <expr with loop var> for <loop var> in <sequence expr >  
if <conditional expression with loop var> ]
```


List Comprehensions vs for Loops

- List comprehensions always return a list!
- For loops do not return anything.

```
my_data = []  
for item in range(10):  
    my_data.append(item)  
my_data
```

or

```
my_data = [ item for item in range(10) ]
```

Why use list comprehensions?

- Transforming elements in a list
- Filtering a list
- Combining the two!

This is a *surprising* number of tasks!

Demo!

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Higher Order Functions:
Functions that accept functions as input

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Learning Objectives

- Learn how to use and create higher order functions:
- Functions can be used as data
- **Functions can accept a function as an argument**
- Functions can return a new function

Code is a Form of Data

- Numbers, Strings: All kinds of data
- Code is its own kind of data, too!
- Why?
 - More expressive programs, a new kind of abstraction.
 - "Encapsulate" logic and data into neat packages.
- This will be one of the trickier concepts in CS88.

What is a Higher Order Function?

- A function that takes in another function as an argument

OR

- A function that returns a function as a result.

Brief Aside: `import`

- Python organizes code in modules
 - These functions come with Python, but you need to "import" them.
- `import module_name`
 - gives us access to `module_name` and `module_name.x`
- `import module_name as my_module`
 - can access `my_module` and `my_module.x` (same code, just a different name)
- `from module_name import x, y, z`
 - can only access the functions we import. `x` is `my_module.x`

```
from math import pi, sqrt
```

```
from operator import mul
```


An Interesting Example

$$\sum_{k=1}^5 k = 1 + 2 + 3 + 4 + 5 = 15$$

$$\sum_{k=1}^5 k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$\sum_{k=1}^5 \frac{8}{(4k-3) \cdot (4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} = 3.04$$

Why Higher Order Functions?

- We can sum 1 to N easily enough.
- We can sum 1 to N^2 easily enough too.
- Or we can sum, 1 to N^3 ...
- But why write so many functions?

Why not write *one function(!)* which allows us flexibility in solving many problems?

A Generic Sum Function

```
def summation(n, term_fn):  
    """Sum the first N terms of a sequence.  
    >>> summation(5, cube)  
    225  
    >>> summation(5, identity)  
    15  
    >>> summation(10, identity)  
    55  
    """  
    total = 0  
    for i in range(n + 1):  
        total = total + term_fn(i)  
    return total
```

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Higher Order Functions:
Functions that return another function

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Higher Order Functions

- **A function that returns (makes) a function**

```
def leq_maker(c):  
    def leq(val):  
        return val <= c  
    return leq
```

```
>>> leq_maker(3)  
<function leq_maker.<locals>.leq at 0x1019d8c80>
```

```
>>> leq_maker(3)(4)  
False
```

```
>>> leq_fn = leq_maker(3)  
>>> leq_fn(4)  
False
```

```
>>> [x for x in range(7) if leq_maker(3)(x)]  
[0, 1, 2, 3]
```


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Environments & Higher Order Functions

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Environment Diagram motivation: variable aliasing

- With HOF's, at first glance there can be confusion between local function variables, nested function variables, and global variables
- Example: here, there are two `c` variables, and two `val` variables.
 - How do they relate to each other?
 - aka "variable aliasing"

```
def leq_maker(c):  
    def leq(val):  
        return val <= c  
    return leq  
  
c = 4  
val = 2  
leq_fn = leq_maker(c)  
print(f"(v1) {leq_fn(2)}")  
  
c = 1  
# does leq_fn()'s behavior change?  
print(f"(v2) {leq_fn(2)}")
```

Question: what does Python output?

Environment Diagram motivation: variable aliasing

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print(f"(v1) {leq_fn(2)}")  
  
c = 1  
# does leq_fn()'s behavior change?  
print(f"(v2) {leq_fn(2)}")  
(v1) True  
(v2) True
```

Environment Diagram motivation: variable aliasing

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- Example: here, there are two `c` variables, and two `val` variables.
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    def leq(val):  
        return val <= c  
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```

```
c = 4  
val = 2  
leq_fn = leq_maker(c)  
print(f"(v1) {leq_fn(2)}")
```

```
c = 1  
# does leq_fn()'s behavior change?  
print(f"(v2) {leq_fn(2)}")  
(v1) True  
(v2) True
```

Environment Diagram motivation: variable aliasing

- Keeping track of each function's local variables can be tricky.
- **General tip:** drawing pictures is often a helpful strategy for understanding
- Thus: environment diagrams

```
def leq_maker(c):  
    def leq(val):  
        return val <= c  
    return leq
```

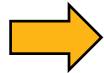
```
c = 4  
val = 2  
leq_fn = leq_maker(c)  
print(f"(v1) {leq_fn(2)}")
```

```
c = 1  
# does leq_fn()'s behavior change?  
print(f"(v2) {leq_fn(2)}")  
(v1) True  
(v2) True
```

Environment Diagram: First Look

(currently evaluating this function call

```
Python 3.6  
(known limitations)  
-----  
1 def leq_maker(c):  
2     def leq(val):  
3         return val <= c  
4     return leq  
5 c = 4  
6 val = 2  
7 leq_fn = leq_maker(c)  
8 print(f"(v1) {leq_fn(2)}")  
9  
10 c = 1  
11 # does leq_fn()'s behavior change?  
12 print(f"(v2) {leq_fn(2)}")  
-----
```



[Edit this code](#)

→ line that just executed
→ next line to execute

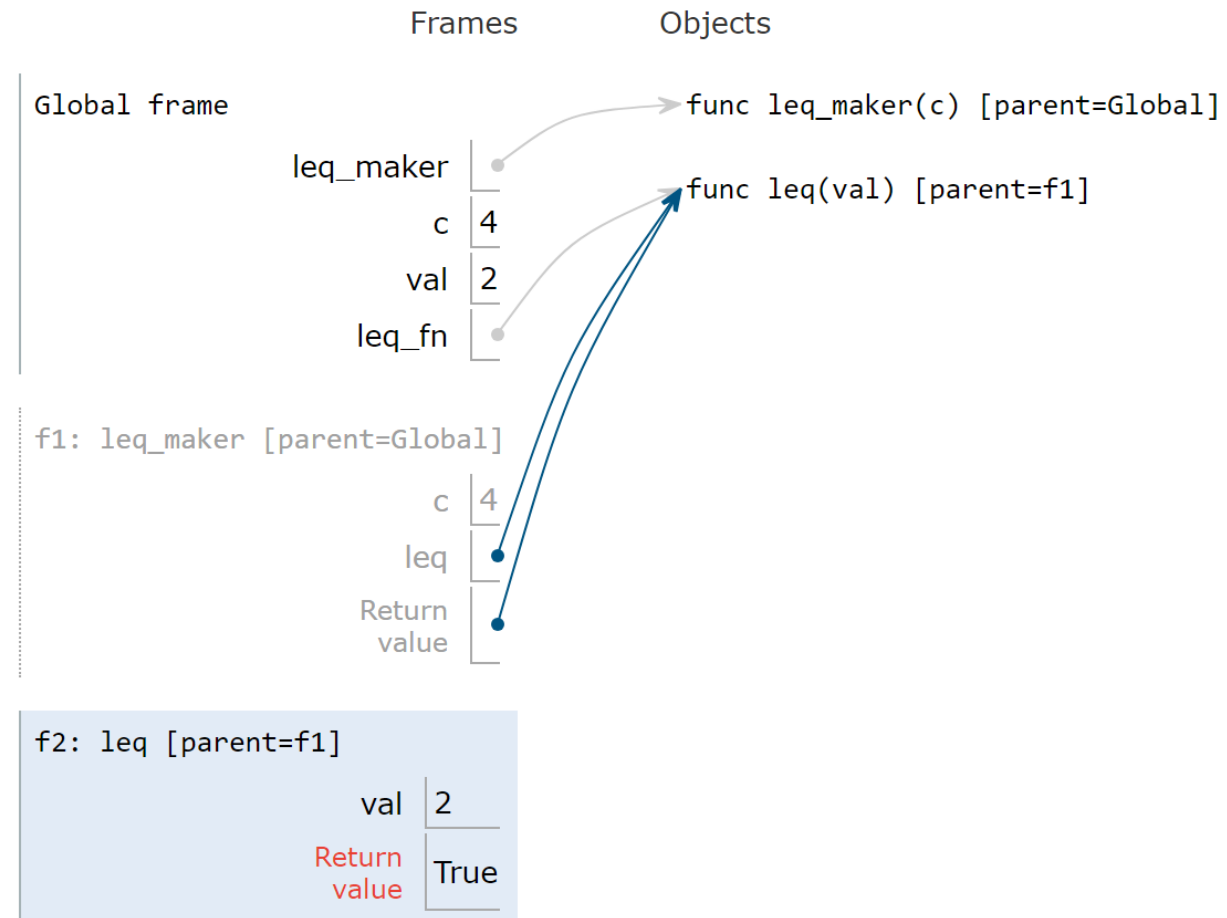


<< First < Prev Next > Last >>

Step 12 of 17

[Customize visualization](#)

Print output (drag lower right corner to resize)

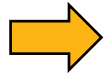


([Python tutor link](#))

Environment Diagram: First Look: Frames

(currently evaluating this function call

```
Python 3.6  
(known limitations)  
1 def leq_maker(c):  
2     def leq(val):  
3         return val <= c  
4     return leq  
5 c = 4  
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8 print(f"(v1) {leq_fn(2)}")  
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11 # does leq_fn()'s behavior change?  
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```



[Edit this code](#)

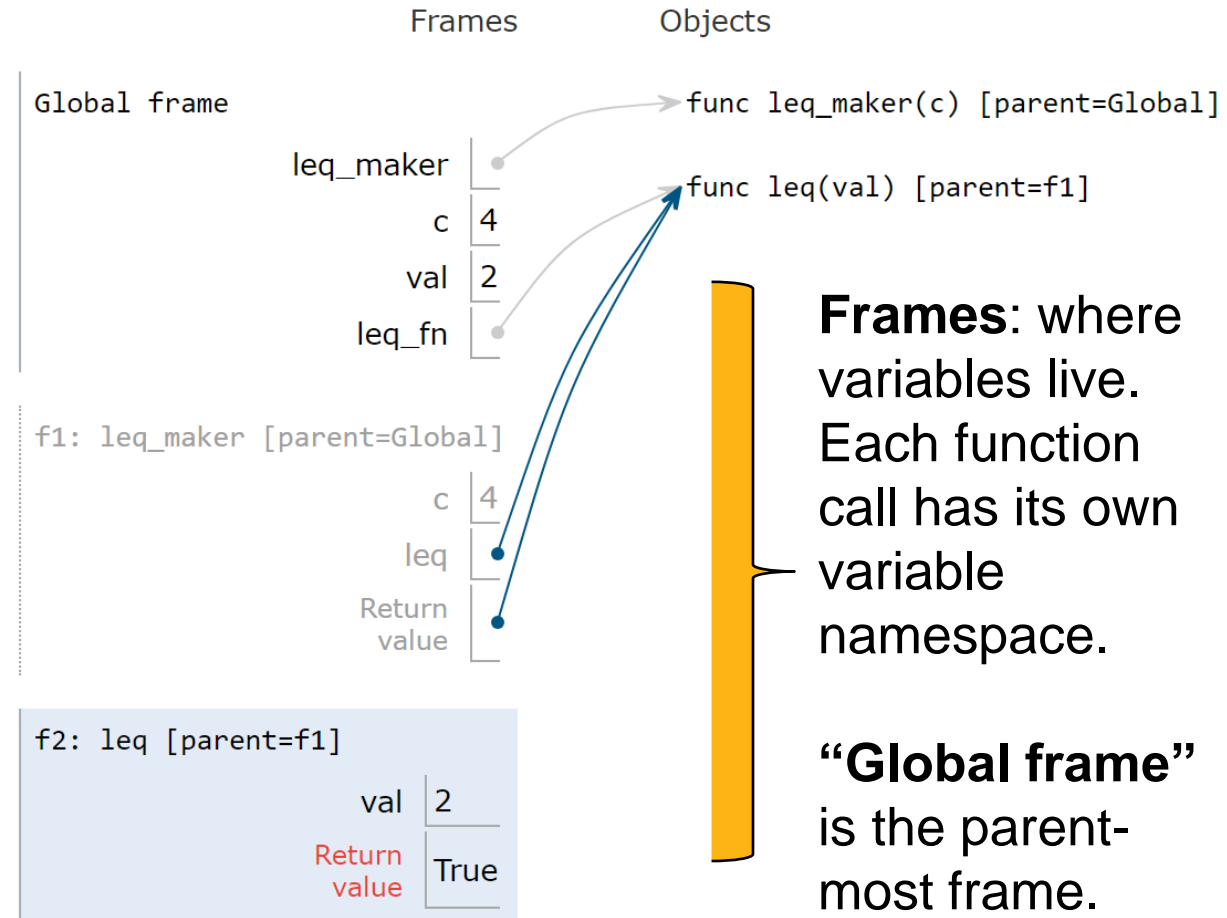
→ line that just executed
→ next line to execute

Progress bar with navigation buttons: << First, < Prev, Next >, Last >>

Step 12 of 17

[Customize visualization](#)

Print output (drag lower right corner to resize)



[\(Python tutor link\)](#)

Environment Diagram: First Look: Frames

Python 3.6
([known limitations](#))

```
1 def leq_maker(c):  
2     def leq(val):  
3         return val <= c  
4     return leq  
5 c = 4  
6 val = 2  
7 leq_fn = leq_maker(c)  
8 print(f"(v1) {leq_fn(2)}")  
9  
10 c = 1  
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```

[Edit this code](#)

→ line that just executed
→ next line to execute

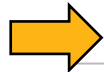


<< First < Prev Next > Last >>

Step 17 of 17

[Customize visualization](#)

(currently evaluating this function call



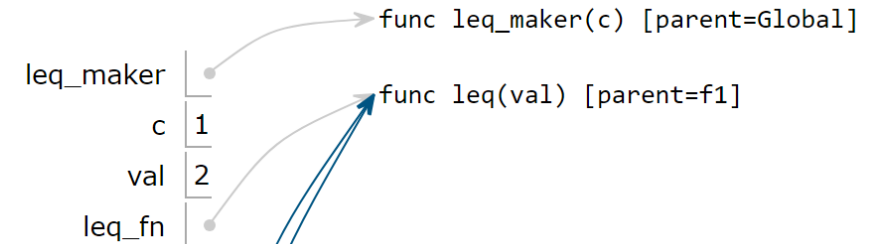
Print output (drag lower right corner to resize)

(v1) True

Frames

Objects

Global frame



f1: leq_maker [parent=Global]

c → 4
leq →
Return value →

f2: leq [parent=f1]

val → 2
Return value → True

f3: leq [parent=f1]

val → 2
Return value → True

Rule: each time you evaluate a **function call**, you **draw a new frame**

Each frame has a **“parent frame”**, dictates variable resolution order

Ex: in leq_fn, `c` evaluates to 4, NOT 1

Example: compose

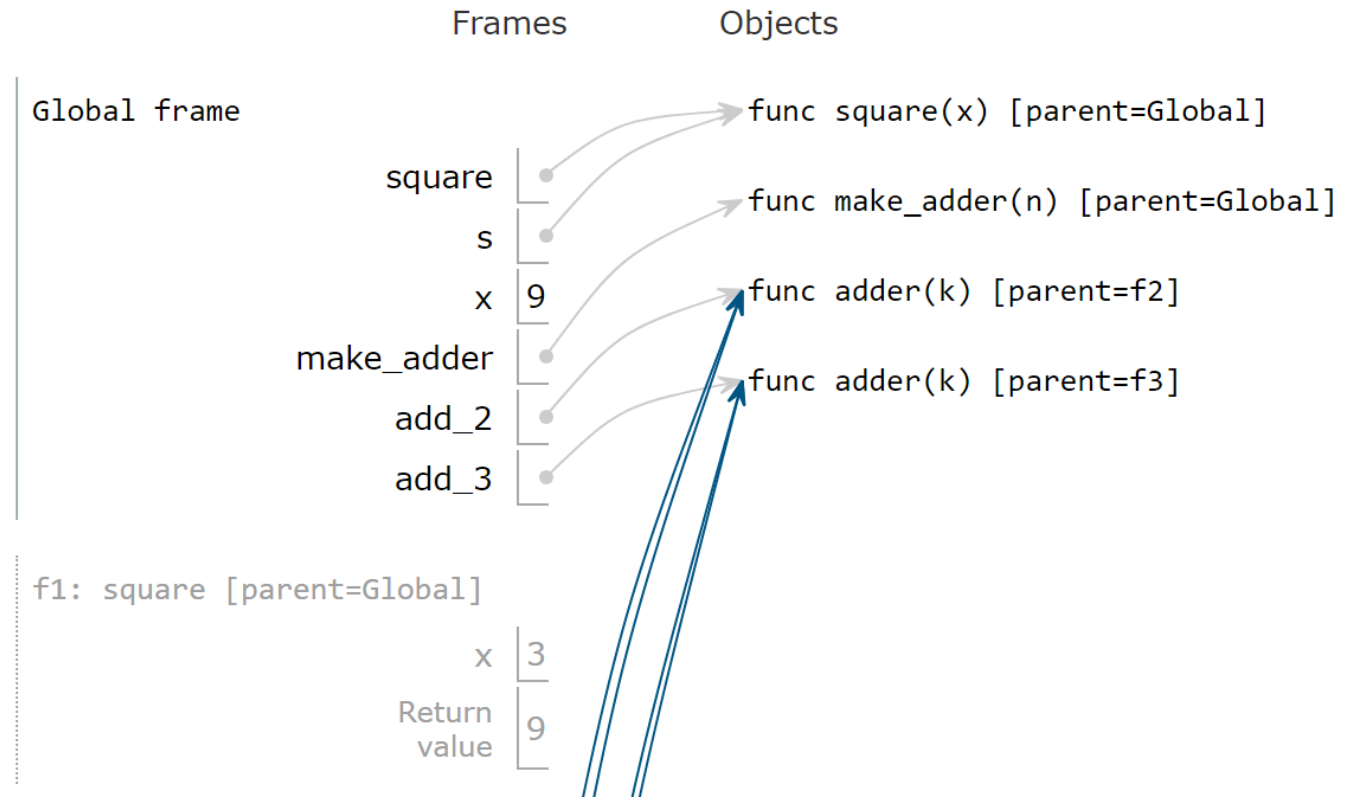
- Python Tutor is a handy web tool that allows you to visualize the environment diagrams of your own Python code! Useful study tool.

- Example:

```
http://pythontutor.com/composingprograms.html#code
=def%20square%28x%29%3A%0A%20%20%20%20return%20x%2
0*%20x%0A%20%20%20%20%0As%20%3D%20square%0Ax%20%3D
%20s%283%29%0A%0Adef%20make_adder%28n%29%3A%0A%20%
20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20
%20%2
```

Environment Diagrams

- Organizational tools that help you understand code
- Allows us to more-precisely define how Python evaluates code
 - Up until now, we've been somewhat hand-wavy



Environment Diagrams: Terminology

- Organizational tools that help you understand code
- **Terminology:**
 - **Frame:** keeps track of variable-to-value bindings, each function call has a frame
 - **Global Frame:** global for short, the starting frame of all python programs, doesn't correspond to a specific function
 - **Parent Frame:** The frame of where a function is defined (default parent frame is global)
 - **Frame number:** What we use to keep track of frames, f1, f2, f3, etc
 - **Variable vs Value:** $x = 1$. x is the **variable**, 1 is the **value**

Environment Diagrams Steps

1. Draw the global frame
2. When evaluating assignments (lines with single equal), always evaluate right side first
3. When you call a function MAKE A NEW FRAME!
4. When assigning a primitive expression (number, boolean, string) write the value in the box
5. When assigning anything else, draw an arrow to the value
6. When calling a function, name the frame with the intrinsic name – the name of the function that variable points to
7. The parent frame of a function is the frame in which it was defined in (default parent frame is global)
8. If the value isn't in the current frame, search in the parent frame

Environment Diagram Tips / Links

- NEVER EVER draw an arrow from one variable to another.
- Useful Resources:
 - [http://markmiyashita.com/cs61a/environment diagrams/rules of environment diagrams/](http://markmiyashita.com/cs61a/environment%20diagrams/rules%20of%20environment%20diagrams/)
 - <http://albertwu.org/cs61a/notes/environments.html>
- **Tip:** historically, students have had trouble with drawing environment diagrams (eg on exams). Let's do a great job this semester!

Today's Overview. Any questions?

- List comprehensions
- Higher order functions
- Environment Diagrams