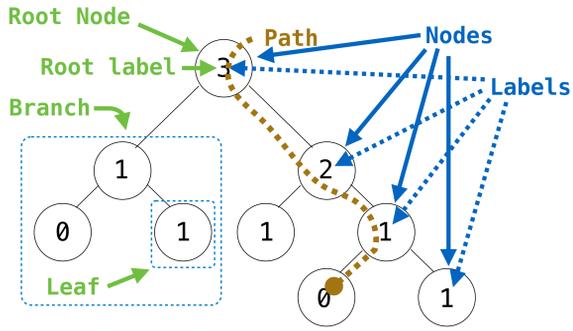


Recursive description:

- A tree has a root label and a list of branches
- Each branch is a tree
- A tree with zero branches is called a leaf

Relative description:

- Each location is a node
- Each node has a label
- One node can be the parent/child of another

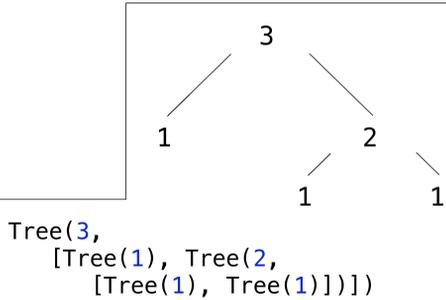


```
class Tree:
    def __init__(self, value, branches=[]):
        self.value = value
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)

    def is_leaf(self):
        return not self.branches

def leaves(tree):
    "The leaf values in a tree."
    if tree.is_leaf():
        return [tree.value]
    else:
        lst = []
        for b in tree.branches:
            lst.extend(leaves(b))
        return lst
```

Built-in `isinstance` function: returns True if `branch` has a class that is or inherits from `Tree`



```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.value + right.value
        return Tree(fib_n, [left, right])
```

Exceptions are raised with a raise statement.

```
raise <expr>
```

<expr> must evaluate to a subclass of BaseException or an

```
try:
    <try suite>
except <exception class> as <name>:
    <except suite>
```

```
>>> try:
    x = 1/0
except ZeroDivisionError as e:
    print('handling a', type(e))
    x = 0

handling a <class 'ZeroDivisionError'>
>>> x
0
```

The <try suite> is executed first.

If, during the course of executing the <try suite>, an exception is raised that is not handled otherwise, and

If the class of the exception inherits from <exception class>, then

The <except suite> is executed, with <name> bound to the exception.

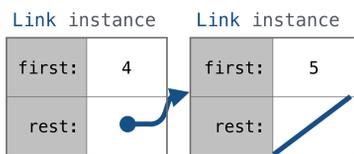
```
class Link:
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest = ',' + repr(self.rest)
        else:
            rest = ''
        return 'Link(' + repr(self.first) + rest + ')'

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + '>'
```

Some zero length sequence



```
>>> s = Link(4, Link(5))
>>> s
Link(4, Link(5))
>>> s.first
4
>>> s.rest
Link(5)
>>> print(s)
<4 5>
>>> print(s.rest)
<5>
>>> s.rest.rest is Link.empty
True
```

Anatomy of a recursive function:

- The **def statement header** is like any function
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**
- Recursive cases are evaluated **with recursive calls**

```
def sum_digits(n):
    "Sum the digits of positive integer n."
    if n < 10:
        return n
    else:
        all_but_last, last = n // 10, n % 10
        return sum_digits(all_but_last) + last
```

- **Recursive decomposition:** finding simpler instances of a problem.
- E.g., `count_partitions(6, 4)`
- Explore two possibilities:
 - Use at least one 4
 - Don't use any 4
- Solve two simpler problems:
 - `count_partitions(2, 4)`
 - `count_partitions(6, 3)`
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Python object system:

Idea: All bank accounts have a **balance** and an account **holder**; the **Account** class should add those attributes to each of its instances

```
>>> a = Account('Jim')
>>> a.holder
'Jim'
>>> a.balance
0
```

An account instance

```
balance: 0 holder: 'Jim'
```

When a class is called:

1. A new instance of that class is created:
2. The `__init__` method of the class is called with the new object as its first argument (named `self`), along with any additional arguments provided in the call expression.

```
class Account:
    def __init__(self, account_holder):
        self.balance = 0
        self.holder = account_holder
    def deposit(self, amount):
        self.balance = self.balance + amount
        return self.balance
    def withdraw(self, amount):
        if amount > self.balance:
            return 'Insufficient funds'
        self.balance = self.balance - amount
        return self.balance
```

```
>>> type(Account.deposit)
<class 'function'>
>>> type(a.deposit)
<class 'method'>
```

```
>>> Account.deposit(a, 5)
10
>>> a.deposit(2)
12
```

Call expression

Dot expression

The `<expression>` can be any valid Python expression. The `<name>` must be a simple name. Evaluates to the value of the attribute looked up by `<name>` in the object that is the value of the `<expression>`.

- To evaluate a dot expression:
1. Evaluate the `<expression>` to the left of the dot, which yields the object of the dot expression
 2. `<name>` is matched against the instance attributes of that object; if an attribute with that name exists, its value is returned
 3. If not, `<name>` is looked up in the class, which yields a class attribute value
 4. That value is returned unless it is a function, in which case a bound method is returned instead

Assignment statements with a dot expression on their left-hand side affect attributes for the object of that dot expression

- If the object is an instance, then assignment sets an instance attribute
- If the object is a class, then assignment sets a class attribute

```
>>> jim_account = Account('Jim')
>>> tom_account = Account('Tom')
>>> tom_account.interest
0.02
>>> jim_account.interest
0.02
>>> Account.interest = 0.04
>>> tom_account.interest
0.04
>>> jim_account.interest
0.04
```

```
>>> jim_account.interest = 0.08
>>> jim_account.interest
0.08
>>> tom_account.interest
0.04
>>> Account.interest = 0.05
>>> tom_account.interest
0.05
>>> jim_account.interest
0.08
```

```
class CheckingAccount(Account):
    """A bank account that charges for withdrawals."""
    withdraw_fee = 1
    interest = 0.01
    def withdraw(self, amount):
        return Account.withdraw(self, amount + self.withdraw_fee)
        or
        return super().withdraw(amount + self.withdraw_fee)
```

To look up a name in a class:

1. If it names an attribute in the class, return the attribute value.
2. Otherwise, look up the name in the base class, if there is one.

```
>>> ch = CheckingAccount('Tom') # Calls Account.__init__
>>> ch.interest # Found in CheckingAccount
0.01
>>> ch.deposit(20) # Found in Account
20
>>> ch.withdraw(5) # Found in CheckingAccount
14
```



```
iter(iterable):
    Return an iterator
    over the elements of
    an iterable value
next(iterator):
    Return the next element

>>> s = [3, 4, 5]
>>> t = iter(s)
>>> next(t)
3
>>> next(t)
4

>>> d = {'one': 1, 'two': 2, 'three': 3}
>>> k = iter(d)
>>> next(k)
'one'
>>> next(k)
'two'
```

A *generator function* is a function that *yields* values instead of *returning* them.

```
>>> def plus_minus(x):
...     yield x
...     yield -x

>>> t = plus_minus(3)
>>> next(t)
3
>>> next(t)
-3

def a_then_b(a, b):
    yield from a
    yield from b

>>> list(a_then_b([3, 4], [5, 6]))
[3, 4, 5, 6]
```

Efficiency

Constant growth: $O(1)$

E.g., accessing a value from a dictionary
Increasing n doesn't affect time

Logarithmic growth: $O(\log(n))$

E.g., binary search
Doubling n only increments time by a constant

Linear growth: $O(n)$

E.g., iterating over a list of length n
Incrementing n increases time by a constant

Quadratic growth: $O(n^2)$

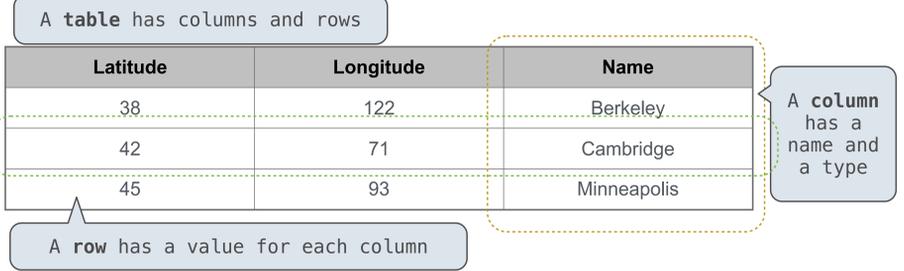
E.g., finding all pairs of a list of integers (double for loop)
Incrementing n increases time by n times a constant

Exponential growth: $O(b^n)$

E.g., recursive fib
Incrementing n multiplies time by a constant

```
def perms(lst):
    """
    Generates the permutations of lst one by one.
    """
    >>> perms = perms([1, 2, 3])
    >>> p = list(perms)
    >>> p.sort()
    >>> p
    [[1, 2, 3], [1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1]]

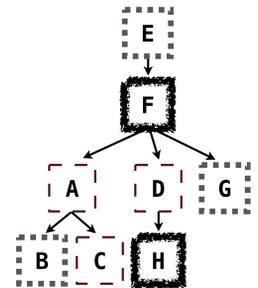
    if lst == []:
        yield []
    else:
        for perm in perms(lst[1:]):
            for i in range(len(lst)):
                yield perm[:i] + [lst[0]] + perm[i:]
```



```
SELECT [expression] AS [name], [expression] AS [name], ... ;
SELECT [columns] FROM [table] WHERE [condition] ORDER BY [order];
```

```
CREATE TABLE parents AS
SELECT "abraham" AS parent, "barack" AS child
UNION
SELECT "abraham"      , "clinton"
UNION
SELECT "delano"       , "herbert"
```

```
CREATE TABLE dogs AS
SELECT "abraham" AS name, "long" AS fur UNION
SELECT "barack"    , "short"  UNION
SELECT "clinton"   , "long"   UNION
SELECT "delano"    , "long"   UNION
SELECT "eisenhower", "short"  UNION
SELECT "fillmore"  , "curly"  UNION
SELECT "grover"    , "short"  UNION
SELECT "herbert"   , "curly";
```



```
SELECT a.child AS first, b.child AS second
FROM parents AS a, parents AS b
WHERE a.parent = b.parent AND a.child <
```

First	Second
barack	clinton
abraham	delano
abraham	grover
delano	grover

The number of groups is the number of unique values of an expression

A *having* clause filters the set of groups that are aggregated

```
select weight/legs, count(*) from animals
group by weight/legs
having count(*)>1;
```

weight/legs	count(*)
5	2
2	2

kind	legs	weight
dog	4	20
cat	4	10
ferret	4	10
parrot	2	6
penguin	2	10
t-rex	2	12000

Annotations for the first table:

- weight/legs=5 (points to row 5, 2)
- weight/legs=2 (points to row 2, 2)
- weight/legs=3 (points to row 2, 2)
- weight/legs=5 (points to row 2, 2)
- weight/legs=6000 (points to row 2, 2)