Welcome to Data C88C!

Lecture 08: Tree Recursion
Thursday, July 3nd, 2025
Week 2
Summer 2025
Instructor: Eric Kim (ekim555@berkeley.edu)

Rev01: 2025-07-04 12:45 PM PST

Announcements

- Lab04, HW04 released today!
  - Due: Tues July 8th, 11:59 PM PST
- Due dates
  - Lab03, HW03 due: Sun July 6th, 11:59 PM PST
- **Add/Drop deadline: Thursday July 3rd**
- Happy 4th of July weekend!

Announcement: Financial Aid Eligibility Survey

"In accordance with federal requirements established by the Department of Education, we need to verify that students are participating in their courses. A survey has been sent to your students in DATA C88C, COMPSCI C88C to confirm their eligibility to receive financial aid. Students will receive separate instructions to complete the 1-question assignment on academic integrity.

You can learn more about the requirement on the Eligibility for Financial Aid at UC Berkeley page."

Students: please check your bCourses for an assignment that verifies your participation in classes. **Required for receiving financial aid**. Read the above link for more info.

Lecture Overview

- Tree Recursion

Recursion Review

How to Know That a Recursive Case is Implemented Correctly

Tracing: Diagram the whole computational process (only feasible for very small examples)

Induction: Check that f(n) is correct as long as f(n-1) ... f(0) are.
(*This the recursive leap of faith.*)

Abstraction: Assume f behaves correctly (on simpler examples), then use it to implement f.

Recall our recursion "motto":

(1) **Divide** – Break the problem down into smaller parts.
(2) **Invoke** – Make the recursive call.
(3) **Combine** – Use the result of the recursive call in your result.
(4) **Base cases** – identify the "smallest" subproblem(s)

Spring 2024 Midterm 1 Question 4(e)

Definition. A *dice integer* is a positive integer whose digits are all from 1 to 6.

```
def streak(n):
    """Return whether positive n is a dice integer in which all the digits are the same.

    >>> streak(22222)
    True
    >>> streak(4)
    True
    >>> streak(22322)  # 2 and 3 are different digits.
    False
    >>> streak(99999)  # 9 is not allowed in a dice integer.
    False
    """
```

(1) **Divide** – Break the problem down into smaller parts.
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Next, try to code it up!

Divide: what is the recursive substructure of `streak`?

Answer: `streak(n)` can be computed by taking the output of `streak(n // 10)` and doing *something* with it.

Combine: given `streak(n // 10)`, how do we solve `streak(n)`?

Answer: `streak(n)` is True iff `streak(n // 10)` is True AND the last (right-most) digit of `n` is the same as the last digit of `n // 10`.

Base cases: what are the smallest subproblems of `streak(n)`?

Answer: single-digit numbers are the base case: return True iff the single-digit number is within [1, 6], False otherwise.

Spring 2024 Midterm 1 Question 4(e)

```
def streak(n):
    """Return whether positive n is a dice integer in which all the digits are the same.

    >>> streak(22222)
    True
    """
    # Base case: single digit. Must be within [1, 6]
    if 1 <= n <= 6:
        return True
    elif n <= 9:
        return False
    # Recursive case.
    # Ex: 2222 is a streak if 222 is a streak AND RHS digit matches
    #   the LHS's digit.
    cur_rhs_digit = n % 10
    next_rhs_digit = (n // 10) % 10
    if cur_rhs_digit != next_rhs_digit:
        return False
    return streak(n // 10)
```

Spring 2024 Midterm 1 Question 4(e)

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    False
    """
    return (n >= 1 and n <= 6) or (n > 9 and n % 10 == n // 10 % 10 and streak(n // 10))
```

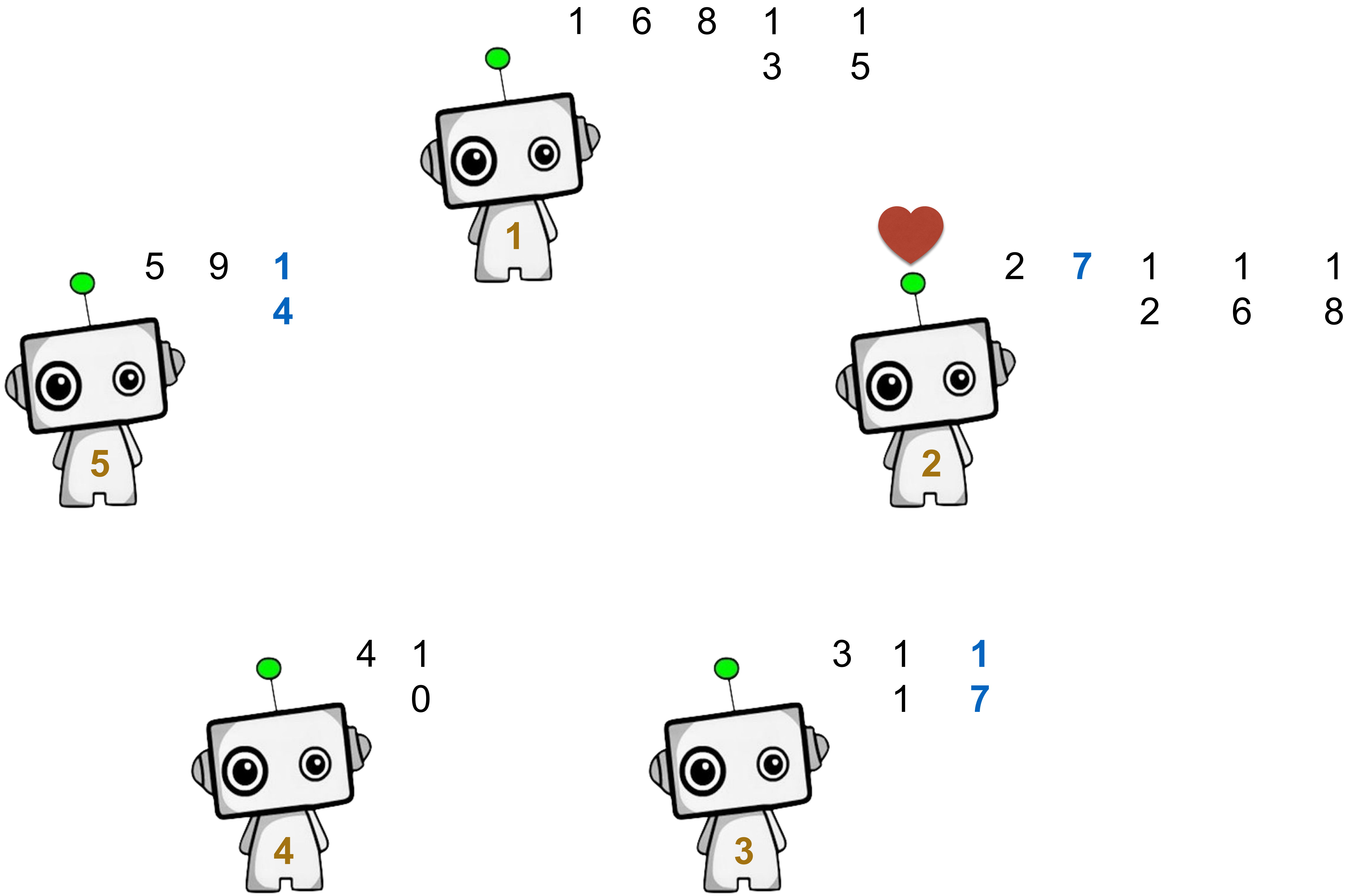
Idea: In a streak, all pairs of adjacent digits are equal.

Another solution, but super compact with aggressive usage of `and` and `or` instead of if statements. See if you can see how this works!

https://pythontutor.com/cp/composingprograms.html#code=def%20streak%28n%29%3A%0A%20%20%20%20return%20%28n%20%3E%3D%201%20and%20n%20%3C%3D%206%29%20or%20%28n%20%3E%209%20and%20n%20%25%2010%20%3D%3D%20n%20//%2010%20%25%2010%20and%20streak%28n%20//%2010%29%29%0A%0Astreak%2822222%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

Discussion Review: Sevens

Players in a circle count up from 1 in the clockwise direction. If a number is divisible by 7 or contains a 7 (or both), switch directions. With 5 players, who says 18?



https://commons.wikimedia.org/wiki/File:Mini-Robot.png; originally from http://kryptonite-bot.ml

The Game of Sevens

Players in a circle count up from 1 in the clockwise direction. If a number is divisible by 7 or contains a 7 (or both), switch directions. If someone says a number when it's not their turn or someone misses the beat on their turn, the game ends.

Implement sevens(n, k) which returns the position of who says n among k players.

1. Pick an example input and corresponding output.
2. Describe a process (in English) that computes the output from the input using simple steps.
3. Figure out what additional names you'll need to carry out this process.
4. Implement the process in code using those additional names.

n: the final number
k: how many players
i: the current number
who: the current player
direction: who's next

(Demo: 08.py:Demo00)

Mutual Recursion

Mutually Recursive Functions:

Two functions f and g are mutually recursive if f calls g and g calls f.

```
def unique_prime_factors(n):
    """Return the number of unique prime factors of n.

    >>> unique_prime_factors(51)  # 3 * 17
    2
    >>> unique_prime_factors(9)   # 3 * 3
    1
    >>> unique_prime_factors(576) # 2 * 2 * 2 * 2 * 2 * 2 * 3 * 3
    2
    """
    k = smallest_factor(n)
    def no_k(n):
        "Return the number of unique prime factors of n other than k."
        if n == 1:
            return 0
        elif n % k != 0:
            return unique_prime_factors(n)
        else:
            return no_k(n // k)
    return 1 + no_k(n)
```

Case 1: we're done dividing out k. Return the number of unique prime factors of the remaining term

Case 2: divide out k

```
def smallest_factor(n):
    "The smallest divisor of n above 1."
```

Approach:
First, identify the smallest factor of `n`.
Then, repeatedly divide out k from n until n is no longer divisible by k.
Then, count the number of unique prime factors of the **remaining term**.

Tip: First, understand what each function's job is (`unique_prime_factors()` vs `no_k()`).

Then, see why they call each other (mutually) to achieve the desired result.

Tree Recursion

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in non-decreasing order.

count_partitions(6, 4)

2 + 4 = 6

1 + 1 + 4 = 6

3 + 3 = 6

1 + 2 + 3 = 6

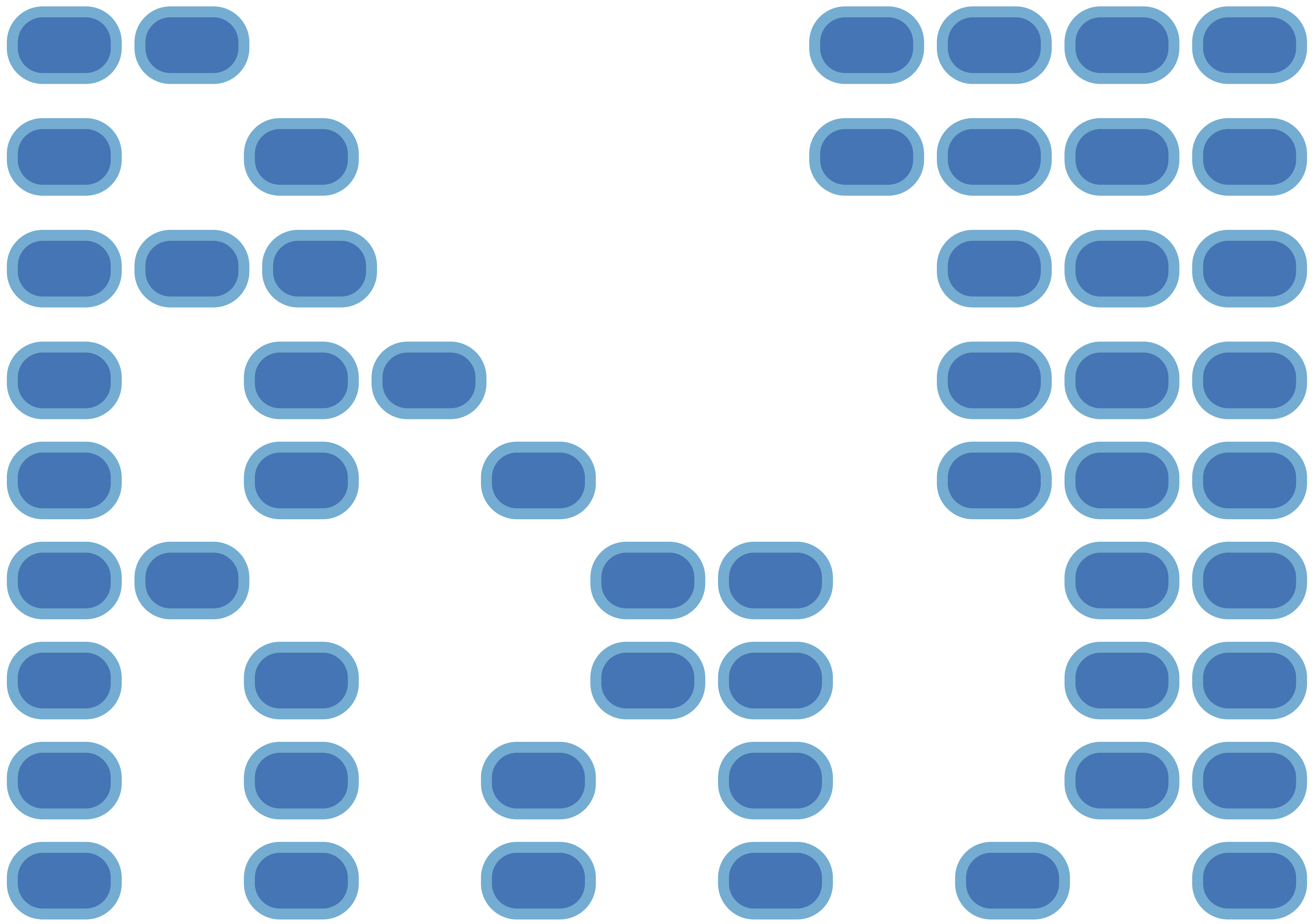
1 + 1 + 1 + 3 = 6

2 + 2 + 2 = 6

1 + 1 + 2 + 2 = 6

1 + 1 + 1 + 1 + 2 = 6

1 + 1 + 1 + 1 + 1 + 1 = 6



Counting Partitions

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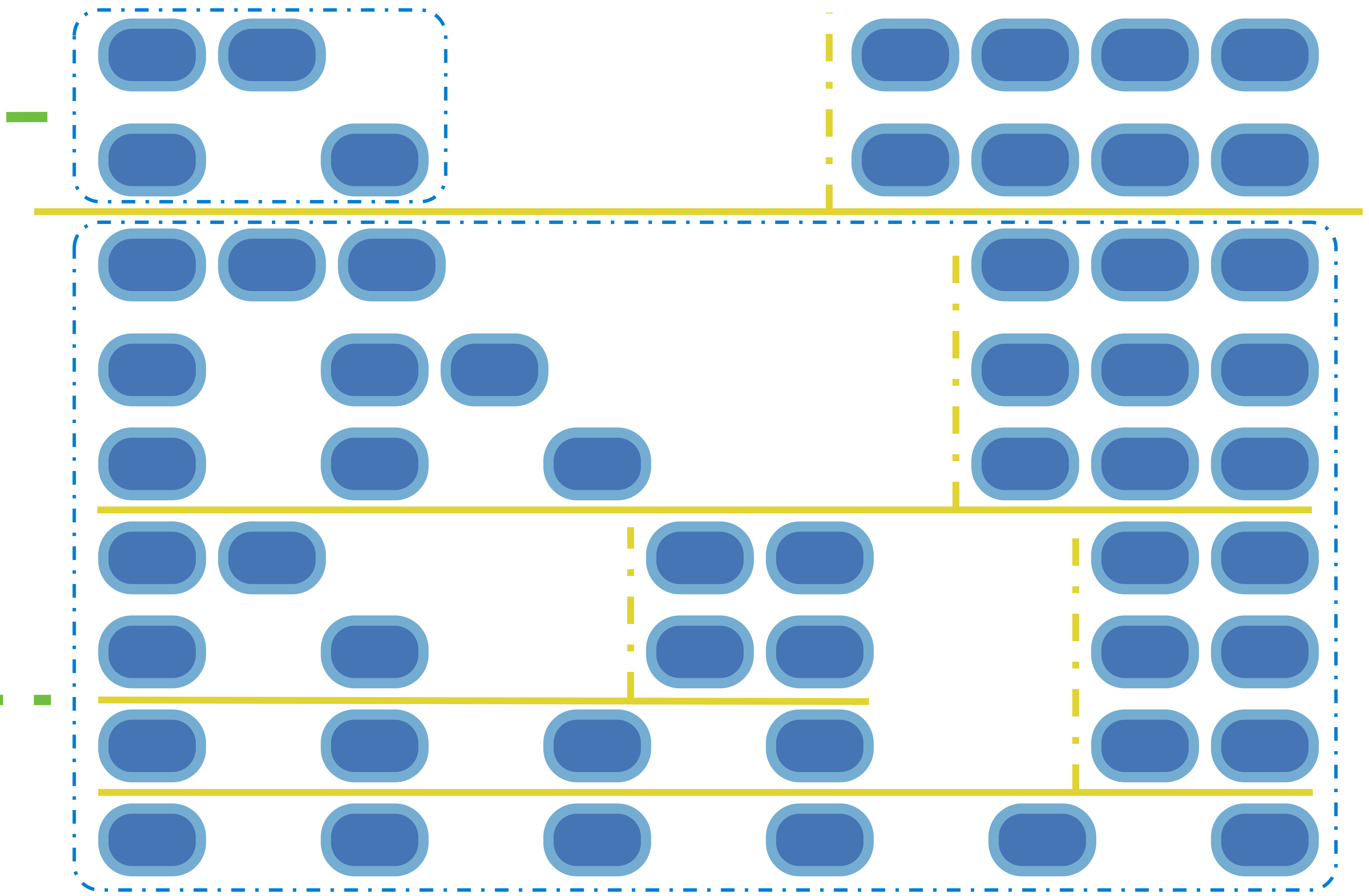
count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.



Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

(Demo)

pythontutor.com/composingprograms.html#code=def%20count_partitions%28n,%20m%29%3A%0A%20%20%20%20if%20n%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%201%0A%20%20%20%20elif%20n%20<%200%3A%0A%20%20%20%20%20%20%20%20return%200%0A%20%20%20%20elif%20m%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%200%0A%20%20%20%20else%3A%0A%20%20%20%20%20%20%20%20with_m%20%3D%20count_partitions%28n-m,%29%20%0A%20%20%20%20%20%20%20%20without_m%20%3D%20count_partitions%28n,%20m-1%29%0A%20%20%20%20%20%20%20%20return%20with_m%20%2B%20without_m%0A%20%20%20%20%20%20%20%20%0Aresult%20%3D%20count_partitions%285,%203%29%0A%0A#%201%20%2B%201%20%2B%201%20%2B%201%20%2B%201%20%3D%205%0A#%201%20%2B%201%20%2B%201%20%2B%202%20%2B%20%20%20%3D%205%0A#%201%20%2B%202%20%2B%202%20%2B%20%220%20%20%20%3D%205%0A#%201%20%2B%201%20%2B%203%20%2B%20%20%20%20%20%20%20%3D%205%0A#%202%20%2B%203%20%2B%20%20%20%20%20%20%20%20%20%20%20%3D%205&mode=display&origin=composingprograms.js&cumulative=false&py=3&rawInputLstJSON=[]&curInstr=0

Spring 2023 Midterm 2 Question 5

Definition. When parking vehicles in a row, a motorcycle takes up 1 parking spot and a car takes up 2 adjacent parking spots. A string of length n can represent n adjacent parking spots using % for a motorcycle, <> for a car, and . for an empty spot.

For example: `'.%%.<><>'` (Thanks to the Berkeley Math Circle for introducing this question.)

Implement **count_park**, which returns the number of ways that vehicles can be parked in n adjacent parking spots for positive integer n. Some or all spots can be empty.

```
def count_park(n):
    """Count the ways to park cars and motorcycles in n adjacent spots.
    >>> count_park(1)  # '.' or '%'
    2
    >>> count_park(2)  # '..', '.%', '%.', '%%', or '<>'
    5
    >>> count_park(4)  # some examples: '<><>', '.%%.', '%<>%', '%.<>'
    29
    """
    if n < 0:
        return ____0_______
    elif n == 0:
        return ____1_______
    else:
        return count_park(n-2) + count_park(n-1) + count_park(n-1)
```

Three choices:

(a) Place a car down (n-2)
(b) Place a motorcycle down (n-1)
(c) Leave an empty space (n-1)