Higher-Order Functions

Discussion 3: June 26, 2025

Call Expressions

Draw an environment diagram for the code below. You can use paper or a tablet or the whiteboard. Then, step through the diagram generated by Python Tutor to check your work.

```
def foo(x, y):
    foo = bar
    return foo(x, y)

def bar(z, x):
    return z + y

y = 5

foo(1, 2)
```

See the web version of this resource for the environment diagram.

Here's a blank diagram in case you're using a tablet:

If you have questions, ask those around you or course staff instead of just looking up the answer!

Global frame	
f1:	[parent=]
	Return Value
f2:	[parent=]
	Return Value

template

Higher-Order Functions

Remember the problem-solving approach from last discussion; it works just as well for implementing higher-order functions.

- 1. Pick an example input and corresponding output. (This time it might be a function.)
- 2. Describe a process (in English) that computes the output from the input using simple steps.
- 3. Figure out what additional names you'll need to carry out this process.
- 4. Implement the process in code using those additional names.
- 5. Determine whether the implementation really works on your original example.
- 6. Determine whether the implementation really works on other examples. (If not, you might need to revise step 2.)

Q1: Make Keeper

Implement make_keeper, which takes a positive integer n and returns a function f that takes as its argument another one-argument function cond. When f is called on cond, it prints out the integers from 1 to n (including n) for which cond returns a true value when called on each of those integers. Each integer is printed on a separate line.

```
def make_keeper(n):
    """Returns a function that takes one parameter cond and prints
    out all integers 1..i..n where calling cond(i) returns True.

>>> def is_even(x): # Even numbers have remainder 0 when divided by 2.
    ... return x % 2 == 0
>>> make_keeper(5)(is_even)
2
4
>>> make_keeper(5)(lambda x: True)
1
2
3
4
5
>>> make_keeper(5)(lambda x: False) # Nothing is printed
"""
"*** YOUR CODE HERE ***"
```

Don't run Python to check your work unless you're confident your answer is correct. You can check it just by thinking!. If you get stuck, ask the staff for help.

Q2: Multi-Apply

Sometimes we want to apply a function more than once to a number. Implement multi-apply, which is a higher-order function takes in a function f. It returns a function of the form g(x, y), which takes in two arguments. This new function *composes*, or applies, f to x y times; for example, for y = 3, it would evaluate f(f(f(x))).

```
def multi_apply(f):
    """Returns a function g(x, y) that returns the result of applying f to x y times.

>>> def adder(x):
    ...    return x + 1
>>> multiadd = multi_apply(adder)
>>> multiadd(3, 1)
4
>>> multiadd(4, 5)
9
>>> multiadd(5, 0)
5
"""
    "*** YOUR CODE HERE ***"
```