Discussion 4: July 3, 2025

Recursion

Many students find this topic challenging. Everything gets easier with practice. Please help each other learn.

Q1: Swipe

Implement swipe, which prints the digits of argument n, one per line, first backward then forward. The left-most digit is printed only once. Do not use while or for or str. (Use recursion, of course!)

```
def swipe(n):
    """Print the digits of n, one per line, first backward then forward.
    >>> swipe(2837)
    7
    3
    8
    2
    8
    3
    7
    if n < 10:
        print(n)
    else:
        print(n % 10)
        swipe(n // 10)
        print(n % 10)
```

Q2: Skip Factorial

Define the base case for the skip_factorial function, which returns the product of every other positive integer, starting with n.

```
def skip_factorial(n):
    """Return the product of positive integers n * (n - 2) * (n - 4) * ...

>>> skip_factorial(5) # 5 * 3 * 1
    15
    >>> skip_factorial(8) # 8 * 6 * 4 * 2
    384
    """

if n <= 2:
    return n
else:
    return n * skip_factorial(n - 2)</pre>
```

Q3: Recursive Hailstone

Recall the hailstone function from Homework 1. First, pick a positive integer n as the start. If n is even, divide it by 2. If n is odd, multiply it by 3 and add 1. Repeat this process until n is 1. Complete this recursive version of hailstone that prints out the values of the sequence and returns the number of steps.

```
def hailstone(n):
    """Print out the hailstone sequence starting at n,
    and return the number of elements in the sequence.
    >>> a = hailstone(10)
    10
    5
    16
    8
    4
    2
    1
    >>> a
    7
   >>> b = hailstone(1)
   >>> b
    1
    0.00
   print(n)
    if n % 2 == 0:
        return even(n)
    else:
        return odd(n)
def even(n):
    return 1 + hailstone(n // 2)
def odd(n):
    if n == 1:
        return 1
    else:
        return 1 + hailstone(3 * n + 1)
```

Extra Questions

The questions below are optional but recommended if you would like some extra practice.

Q4: Is Prime

Implement is_prime that takes an integer n greater than 1. It returns True if n is a prime number and False otherwise. Try following the approach below, but implement it recursively without using a while (or for) statement.

```
def is_prime(n):
    assert n > 1
    i = 2
    while i < n:
        if n % i == 0:
            return False
        i = i + 1
    return True</pre>
```

You will need to define another "helper" function (a function that exists just to help implement this one). Does it matter whether you define it within is_prime or as a separate function in the global frame? Try to define it to take as few arguments as possible.

```
def is_prime(n):
    """Returns True if n is a prime number and False otherwise.
    >>> is prime(2)
    True
    >>> is_prime(16)
    False
    >>> is_prime(521)
    True
    0.00
    def check_all(i):
        "Check whether no number from i up to n evenly divides n."
        if i == n:
                        # could be replaced with i > (n ** 0.5)
            return True
        elif n % i == 0:
            return False
        return check_all(i + 1)
    return check_all(2)
```

Define an inner function that checks whether some integer between i and n evenly divides n. Then you can call it starting with i=2:

```
def is_prime(n):
    def f(i):
        if n % i == 0:
            return ____
        elif ____:
            return ____
        else:
            return f(____)
        return f(2)
```

Q5: Function Repeater

Define a function $make_fn_repeater$ which takes in a one-argument function f and an integer x. It should return another function which takes in one argument, another integer. This function returns the result of applying f to x this number of times.

Make sure to use recursion in your solution.

```
def make_func_repeater(f, x):
    """
    >>> incr_1 = make_func_repeater(lambda x: x + 1, 1)
    >>> incr_1(2) #same as f(f(x))
    3
    >>> incr_1(5)
    6
    """
    def repeat(i):
        if i == 0:
            return x
        else:
            return f(repeat(i - 1))
    return repeat
```